Information recall from long-term memory

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Memory retrieval

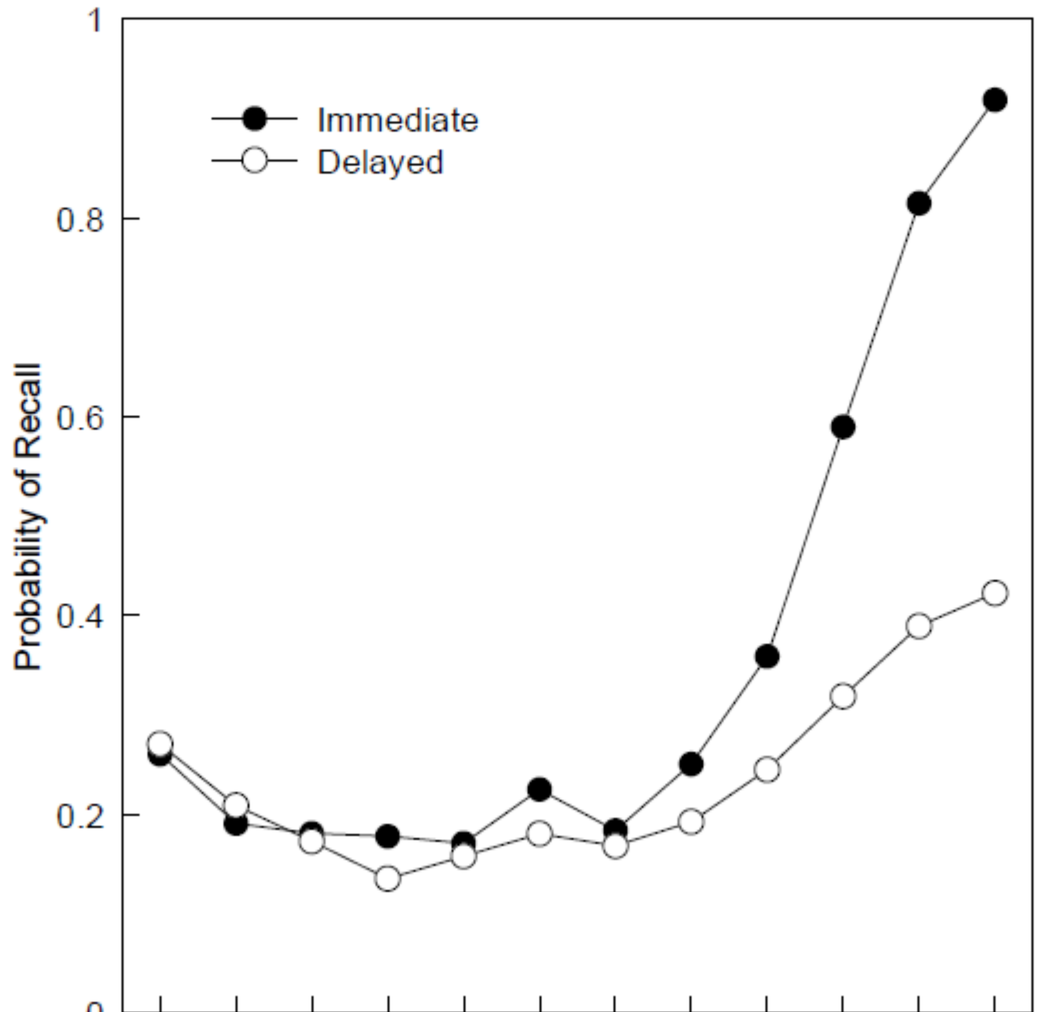
Memory retrieval – without cues

Memory retrieval – with cues

Free Recall

1. Boy
2. Mountain
3. Job
4. Violin
5. Water
6. Plane
7. Monastery
8. Conference
9. Goal
10. Building
11. Bicycle
12. Telephone
13. Suit
14. Defeat
15. Table
16. Engineer

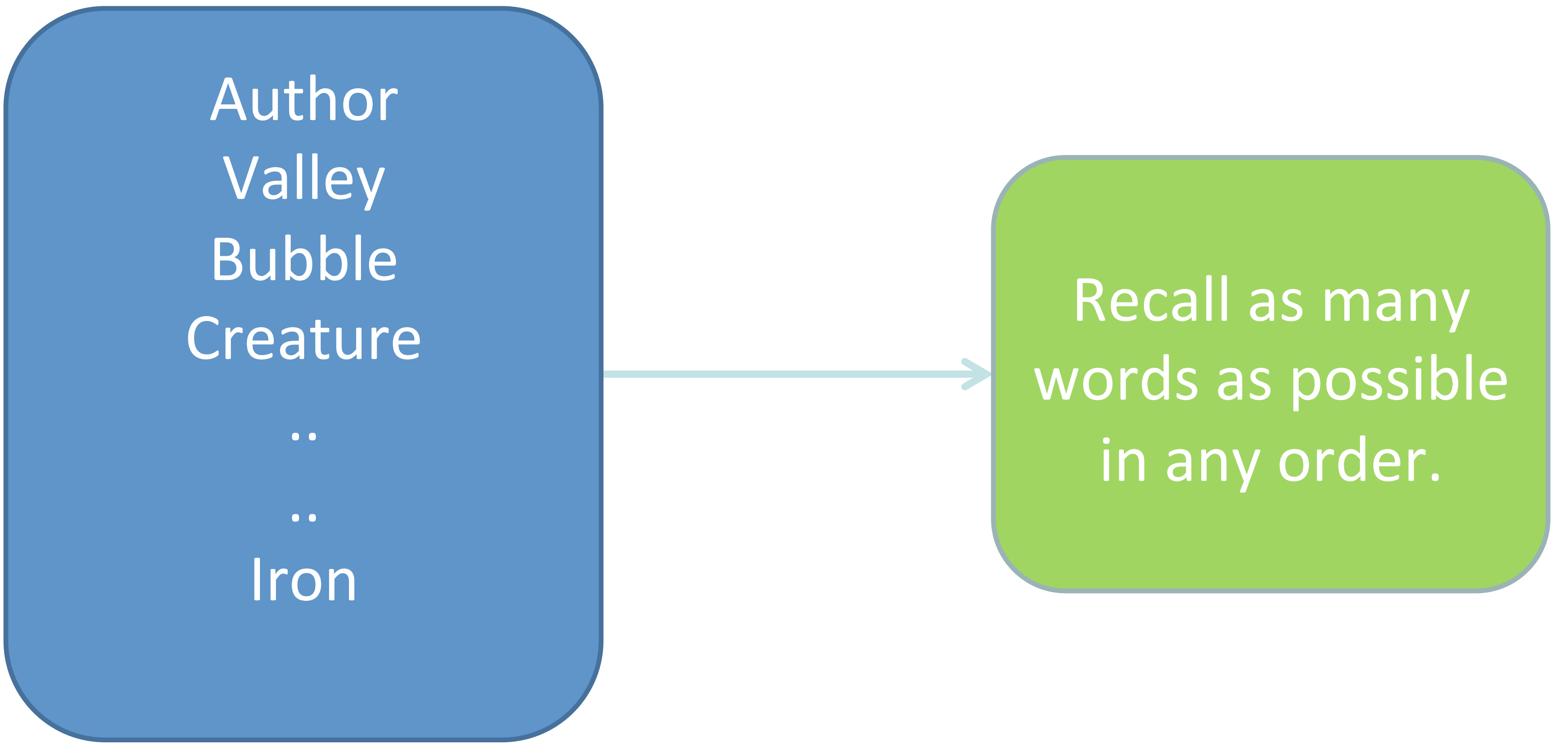
Primacy and recency in free recall



Serial position

Howard & Kahana 1999

Free recall task



Free recall VS Recognition

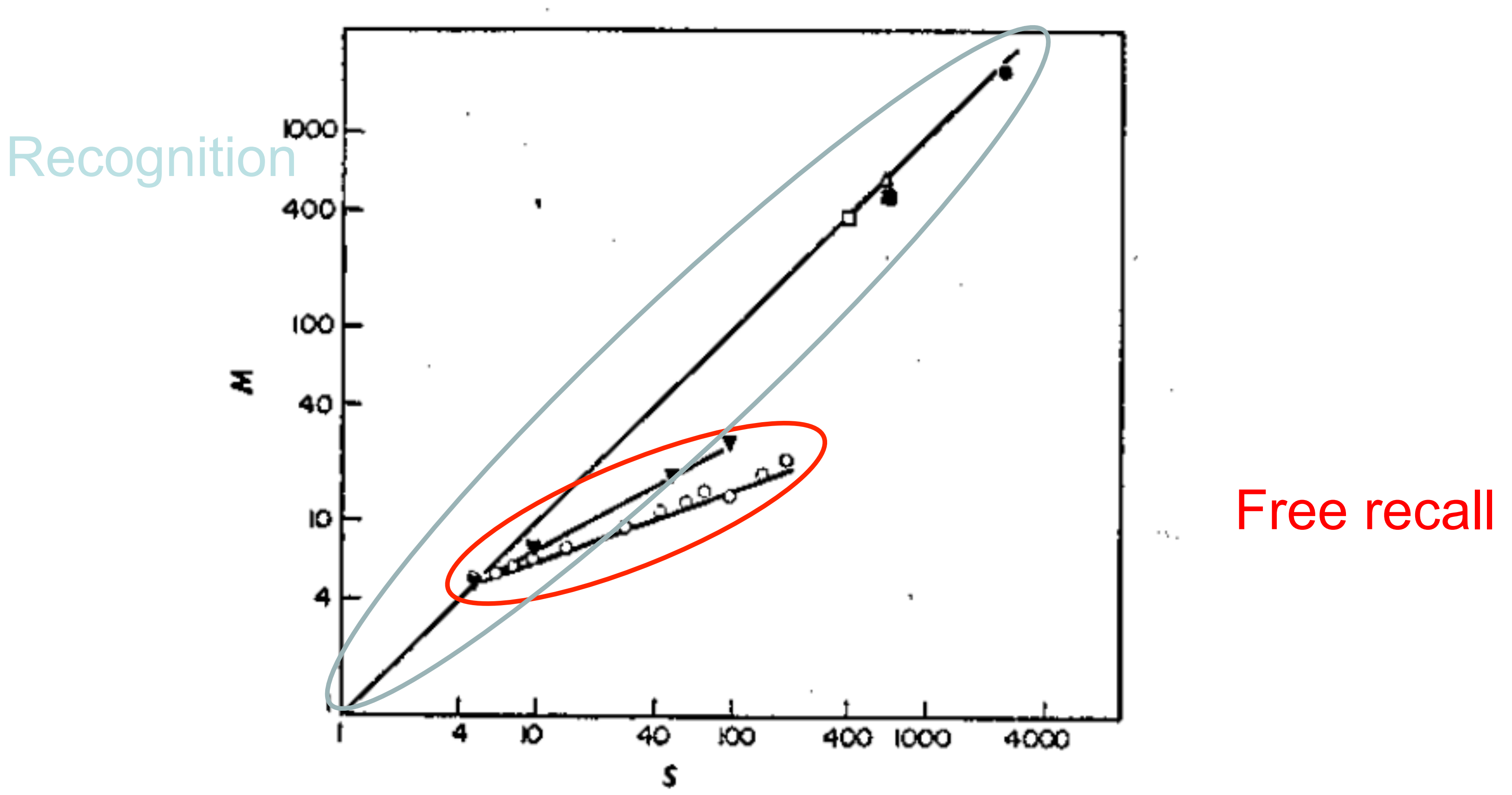


Fig: Standing (1973), *Q J Exp Psy.* Free Recall: Binet & Henri (1894), Murdock (1960) *J Exp Psy*

Journal of Experimental Psychology: Human Learning and Memory
1975, Vol. 104, No. 1, 65–70

Graphemically Cued Retrieval of Words from Long-Term Memory

D. J. Murray, *Queen's University, Kingston, Ontario, Canada*

Subjects were asked to produce in 2 min as many words as they could in which the 1st, 2nd, 3rd, 4th, or 5th letter was A or B or . . . or Z. It was found that the number of words produced was a power function of the number of words we estimated they would know in which the 1st, 2nd, 3rd, 4th, or 5th letter was A or B or . . . or Z (the vocabulary size). Also, with easy retrieval cues, high-frequency words were produced first, which was not the case for difficult retrieval cues. The relationship between word frequency and vocabulary size was also examined.

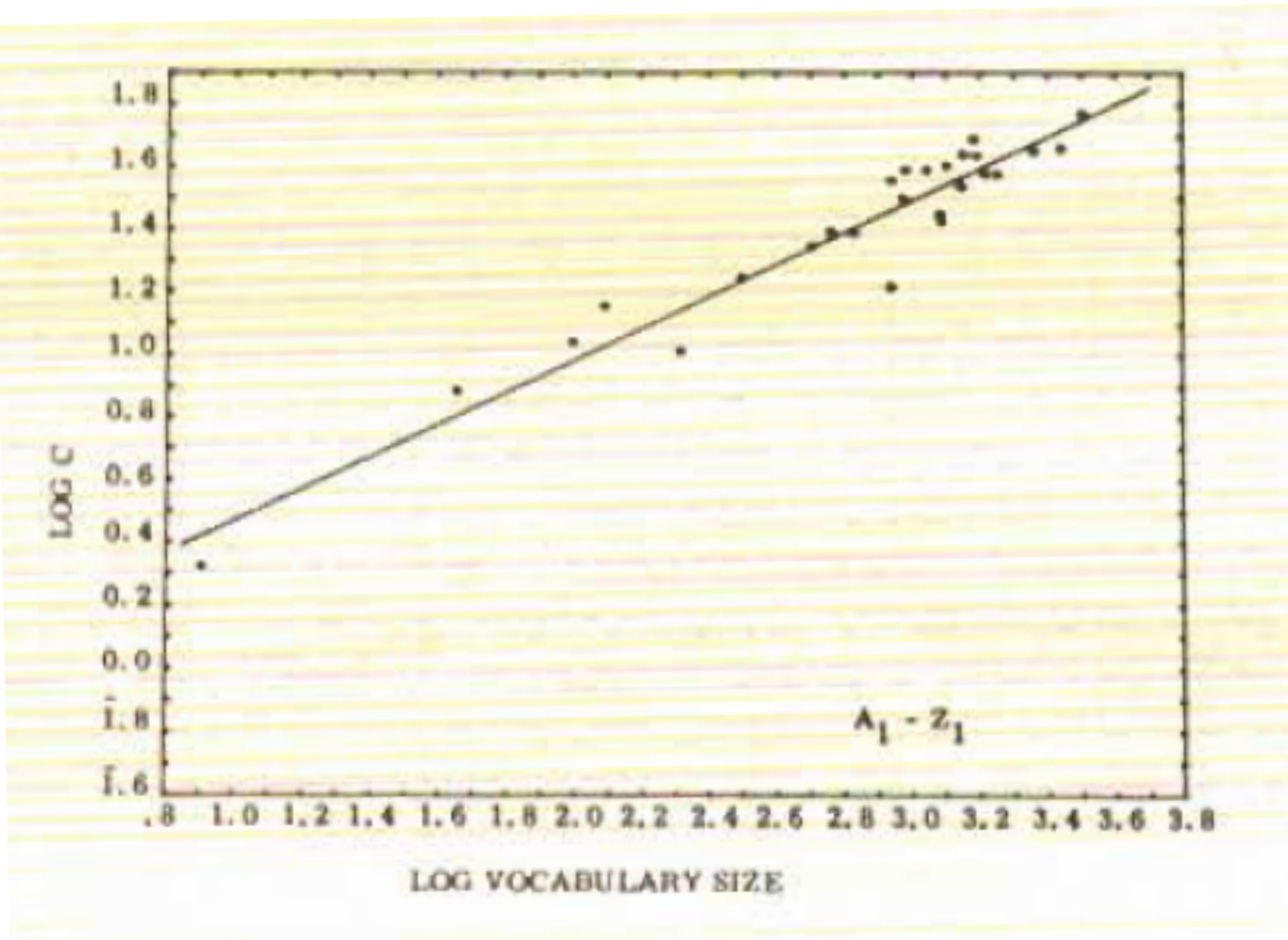
TABLE 1

RESULTS OF THE ANALYSIS OF THORNDIKE-LORGE LISTS

Letter	Cue distance									
	1		2		3		4		5	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
A	1,797	58.13	4,222	19.21	2,466	18.31	1,897	9.45	1,967	6.76
B	1,605	25.17	214	16.73	677	6.12	540	7.31	435	3.29
C	2,761	12.14	429	12.04	1,399	13.04	1,231	12.24	848	9.54
D	1,651	13.33	225	11.00	937	50.92	1,148	15.15	718	18.97
E	1,207	13.69	4,291	22.10	2,065	62.96	3,341	20.78	3,606	14.54
F	1,270	27.32	121	350.03	509	15.61	535	8.05	347	5.42
G	940	14.61	96	23.32	765	12.49	787	9.46	501	15.18
H	1,105	45.60	1,190	114.31	285	10.59	732	28.85	865	15.31
I	1,221	56.64	2,724	22.13	1,674	20.04	2,247	11.57	2,451	9.10
J	316	12.59	10	0.90	75	10.44	50	5.80	10	1.00
K	207	17.28	64	6.53	139	28.55	578	14.08	374	10.75
L	974	17.55	1,301	14.02	1,920	13.44	1,676	16.81	1,702	12.41
M	1,601	19.40	492	11.35	1,180	18.98	849	16.88	684	6.24
N	577	32.51	2,074	45.53	2,423	13.44	1,604	19.88	1,601	11.72
O	676	98.43	4,142	30.18	1,721	23.36	1,573	12.76	1,786	10.40
P	2,251	11.57	527	14.75	1,082	9.21	983	10.08	566	6.71
Q	124	12.50	67	8.18	79	6.16	69	5.39	25	2.80
R	1,401	11.49	2,421	17.34	3,013	19.47	1,952	16.76	2,121	13.67
S	3,188	16.56	282	76.24	1,913	25.00	1,500	17.19	1,360	7.23
T	1,416	107.82	652	41.08	1,845	30.30	2,511	20.44	1,904	12.64
U	872	9.99	2,116	15.48	1,086	20.73	1,038	13.35	897	8.65
V	512	8.96	215	23.47	514	24.56	399	11.58	194	9.49
W	875	65.53	187	23.34	318	32.66	258	20.66	219	6.74
X	7	0.86	284	11.60	124	13.67	71	2.06	35	33.49
Y	99	69.87	326	25.83	286	30.73	260	42.44	549	12.39
Z	45	2.58	16	1.69	93	5.00	96	4.93	53	5.23
Blank	0	—	9	3,164.67	110	1,802.72	722	533.14	2,880	196.50

Note. The words are from Parts 1 and 2.i. of the lists of Thorndike and Lorge (1944). Column (a) is the number of words in which the 1st, 2nd, 3rd, 4th, or 5th letters are A or B or . . . or Z. Column (b) is the mean frequency of words in which the 1st, 2nd, 3rd, 4th, or 5th letters are A or B or . . . or Z. In the latter analysis a value of 0 was given to all words in Part 2.i.

Retrieval from long-term memory – power law

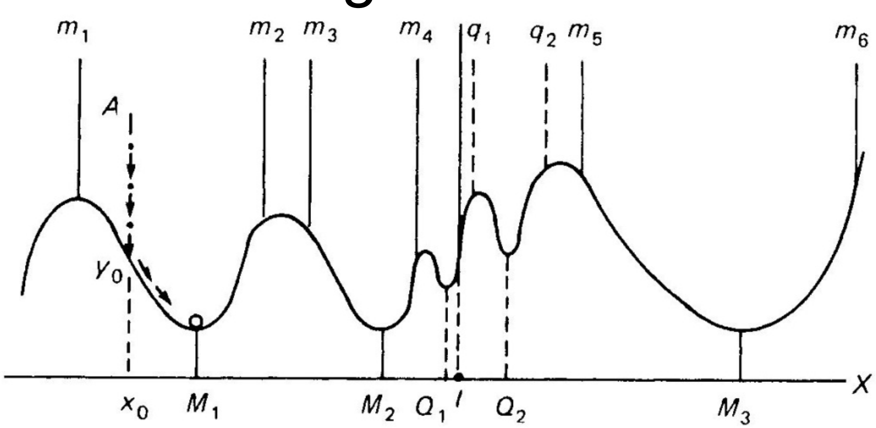


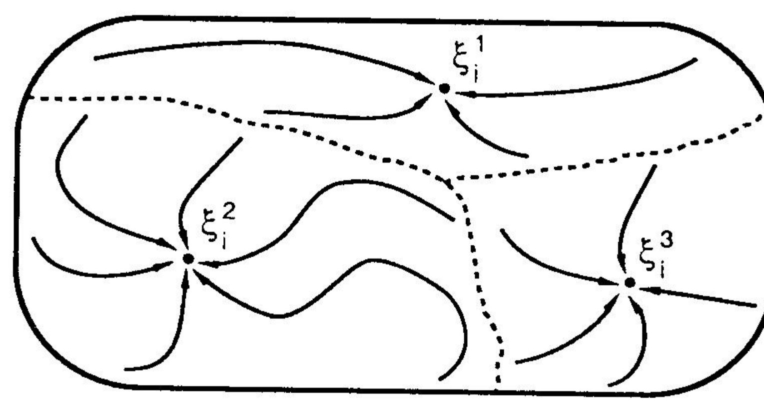
$$C \approx V^{\frac{1}{2}}$$

Neural network models of long-term memory ('Hopfield model')

Memories are represented as attractors (stable states) of network dynamics.

- Attractor = internal representation (memory) of a stimulus
- Each attractor: a subset of neurons that has elevated persistent activity.
- Synaptic changes => Changes in attractor landscape = changes in memory
- Convergence to an attractor = recall of item from memory





Associative memory model (Hopfield 1982)

Neurons (N): $S_i = \pm 1$

$i = 1, \ldots, N$

Connections (N^2): J_{ij}

Network dynamics: $S_i(t+1) = sign(\sum_{j=1}^{N} J_{ij} S_j(t))$

Random memory patterns: $\xi_i^\mu = \pm 1; \quad \mu = 1, \ldots, P$

$$\Pr ob(\xi_i^\mu = 1) = \frac{1}{2}$$

Associative memory model (Hopfield 1982)

Storage:

$$J_{ij}(0) = 0$$

$$\Delta J_{ij}(\mu) = \xi_i^{\mu} \xi_i^{\mu} \Rightarrow$$

$$\mu = 1, \ldots, P$$

$$J_{ij} = \sum_{\mu=1}^{P} \xi_i^{\mu} \xi_j^{\mu}$$

Associative memory model (Hopfield 1982)

Retrieval:

$$S_i(0): \ \xi_i^{\mu}$$

$$S_i(t+1) = sign(\sum_{j=1}^{N} J_{ij} S_j(t))$$

$$\Rightarrow A_i^* = \xi_i^{\mu}$$

Associative memory model (Hopfield 1982)

Capacity:

$$P_{\max} \approx 0.14N$$

Amit et al, Phys. Rev. 1985

Hopfield model with *sparse coding* (Tsodyks, Feigelman 1988)

Neurons (N):

$$V_i = 0,1$$

$$i = 1,\ldots,N$$

Connections (N^2):

$$J_{ij}$$

Network dynamics:

$$V_i(t+1) = St(\sum_{j=1}^{N} J_{ij}V_j(t) - T)$$

***Random* memory patterns :**

$$\xi_i^\mu = 0,1$$

$$\Pr ob(\xi_i^\mu = 1) = f$$

$$f << 1$$

Hopfield model with *sparse coding* (Tsodyks, Feigelman 1988)

Storage:

$$J_{ij}(0) = 0$$

$$\Delta J_{ij}(\mu) = (\xi_i^{\mu} - f)(\xi_i^{\mu} - f) \Rightarrow J_{ij} = \sum_{\mu=1}^{P} (\xi_i^{\mu} - f)(\xi_j^{\mu} - f)$$

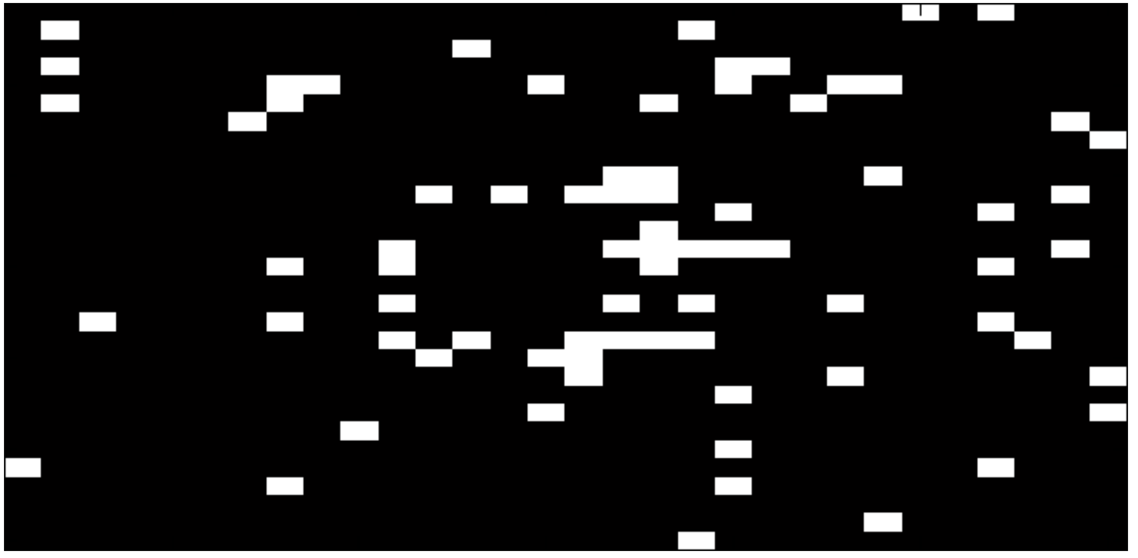
$$\mu = 1, \ldots, P$$

Hopfield model with *sparse coding* (Tsodyks, Feigelman 1988)

Storage capacity:

$$P_{\max} \approx \frac{N}{2f\log(\frac{1}{f})}$$

Cue-triggered retrieval

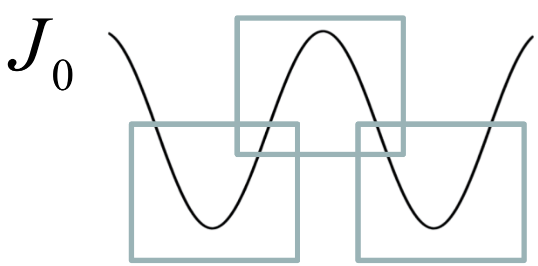


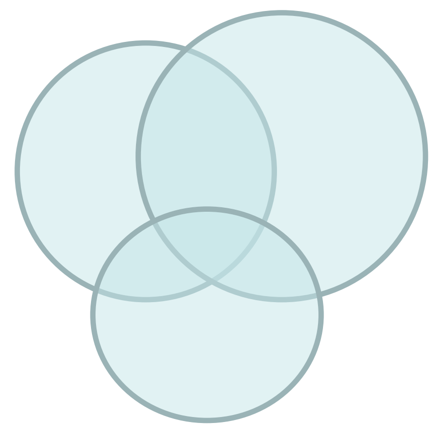
Associative retrieval – attractor neural network model

*) Attractor neural network with sparse coding
   (f*N neurons on average encode each item)
*) Periodic modulation of feedback inhibition
*) Adaptation (Single neuron / Synaptic)

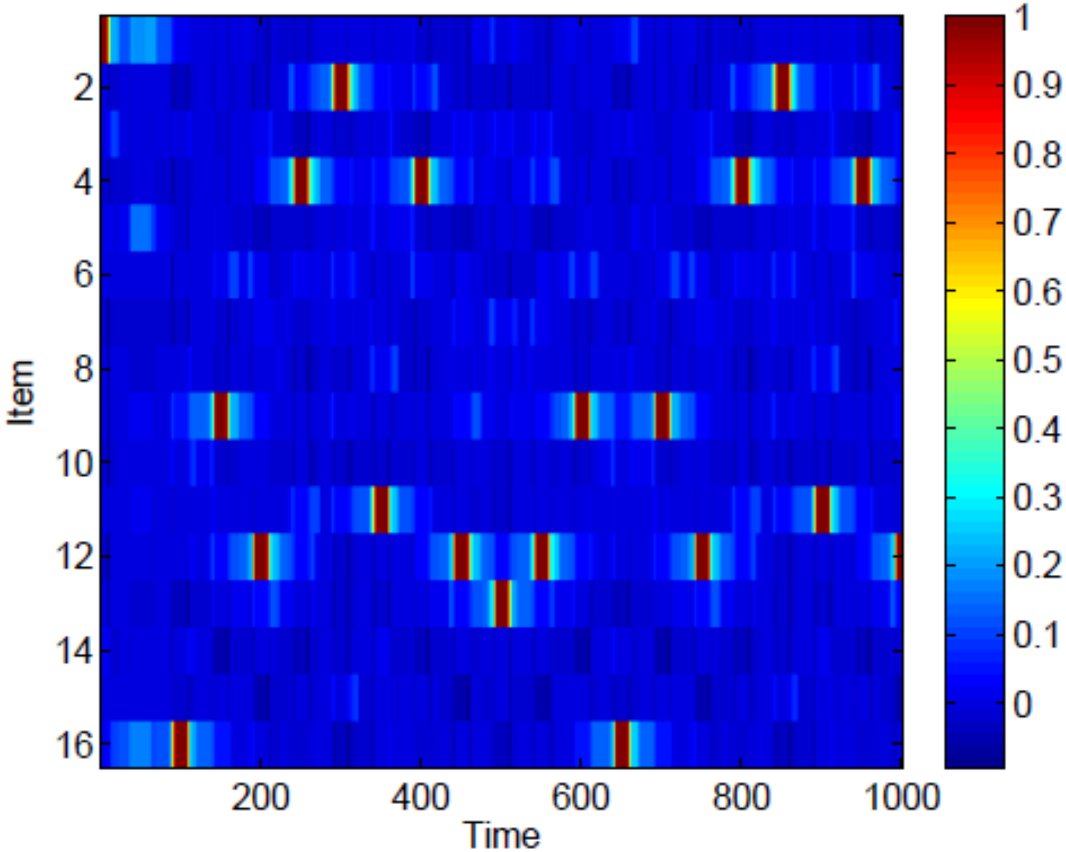
Hebbian learning:

$$J_{ij} = \frac{J_1}{N} \sum_{\mu=1}^{L} (\xi_i^{\mu} - f)(\xi_j^{\mu} - f) - \frac{J_0}{N}$$

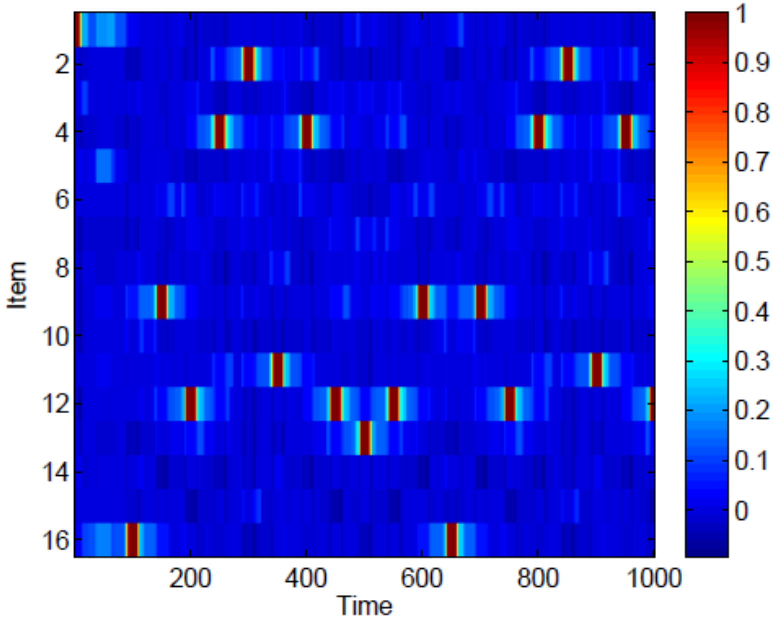




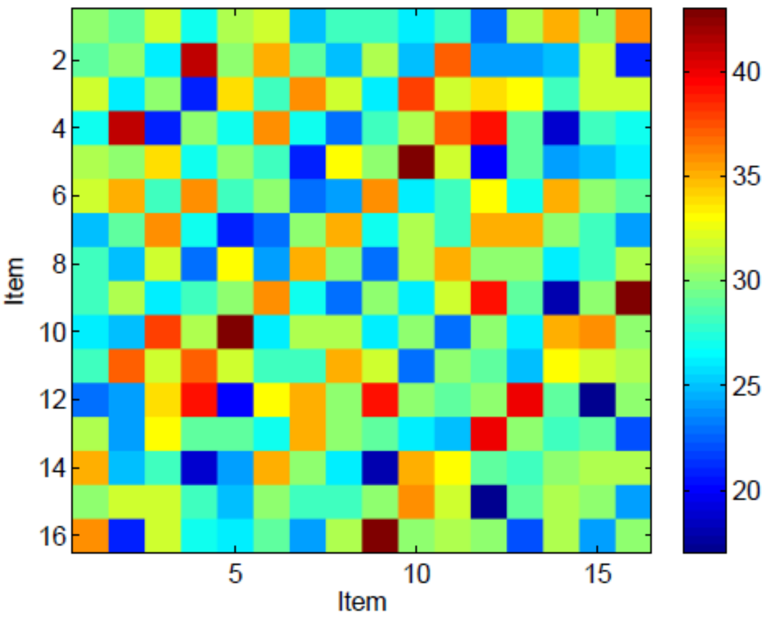
The model – simulations



Transitions are determined by largest intersections



# of neurons in intersections

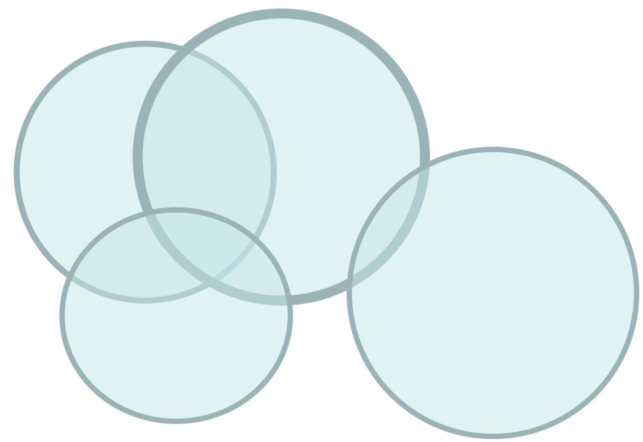


Associative model of free recall

$$\xi_i^{\mu} = (0;1)$$

$i = 1 \ldots N$ Neurons

$\mu = 1 \ldots L$ Items



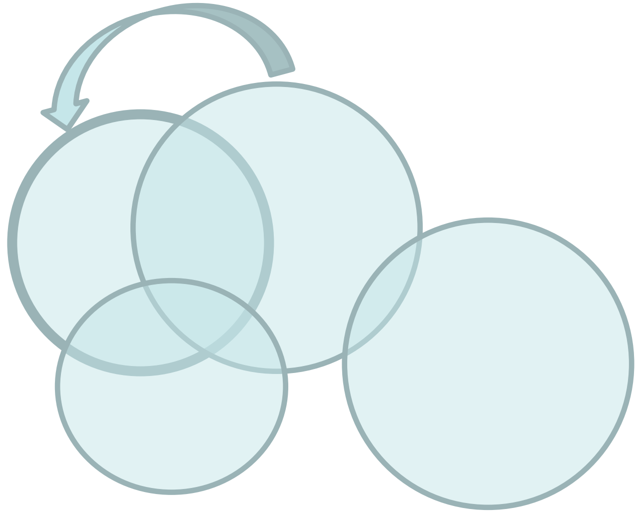
$$\frac{1}{N}\sum_{i=1}^{N} \xi_i^{\mu} \approx f$$

Associative model of free recall

$$\xi_i^{\mu} = (0;1)$$

$i = 1\mathrm{K}\ N$ Neurons

$\mu = 1\mathrm{K}\ L$ Items



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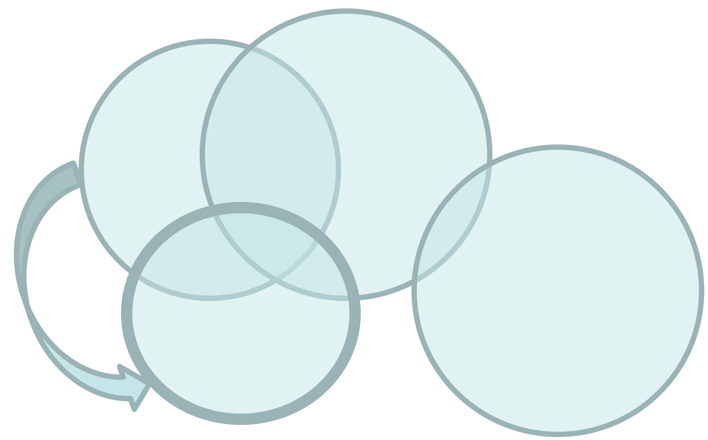

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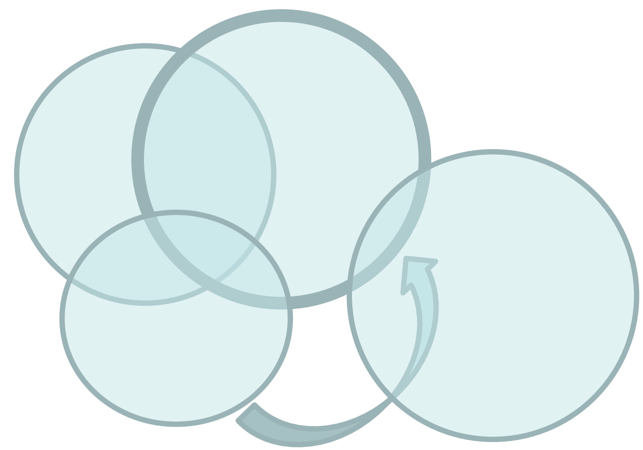

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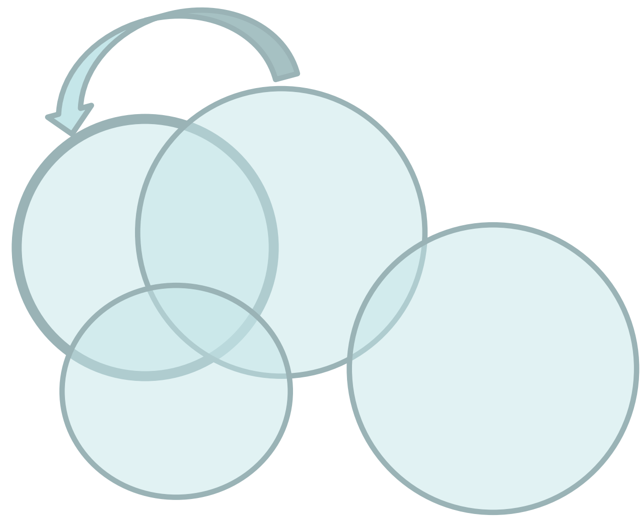

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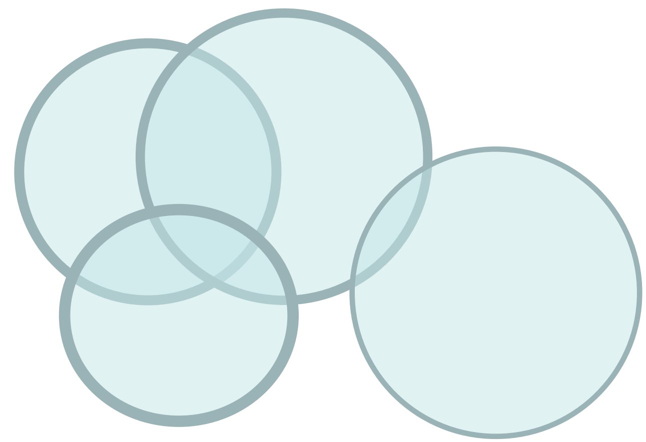
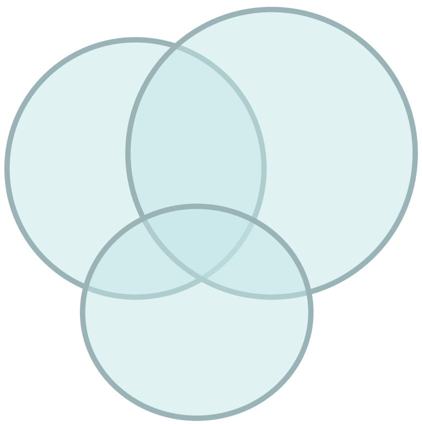


$$\frac{1}{N}\sum_{i=1}^{N} \xi_i^\mu \approx f$$

Similarities as overlaps between neuronal representation

$$S_{\mu,\nu} = \sum_{i=1}^{N} \xi_i^{\mu} \xi_i^{\nu}$$

# of neurons in the intersections between two patterns

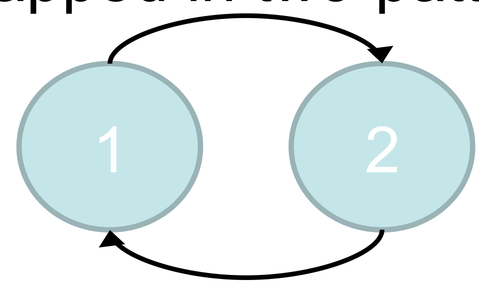


Similarities as overlaps between neuronal representation

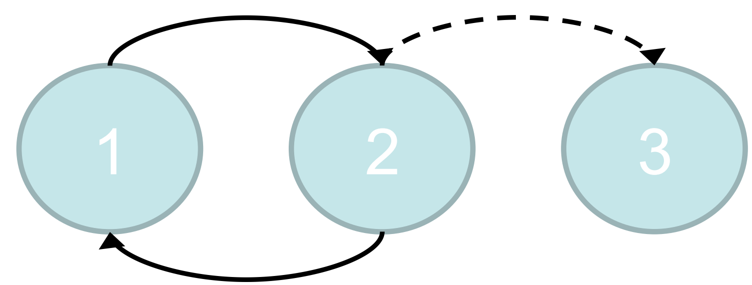
$$S_{\mu,\nu} = \sum_{i=1}^{N} \xi_i^{\mu} \xi_i^{\nu}$$

$$\begin{pmatrix} \times & S_{1,2} & \cdots & S_{1,L} \\ S_{1,2} & \times & \cdots & S_{2,L} \\ \vdots & & \ddots & \vdots \\ S_{1,L} & S_{2,L} & \cdots & \times \end{pmatrix}$$

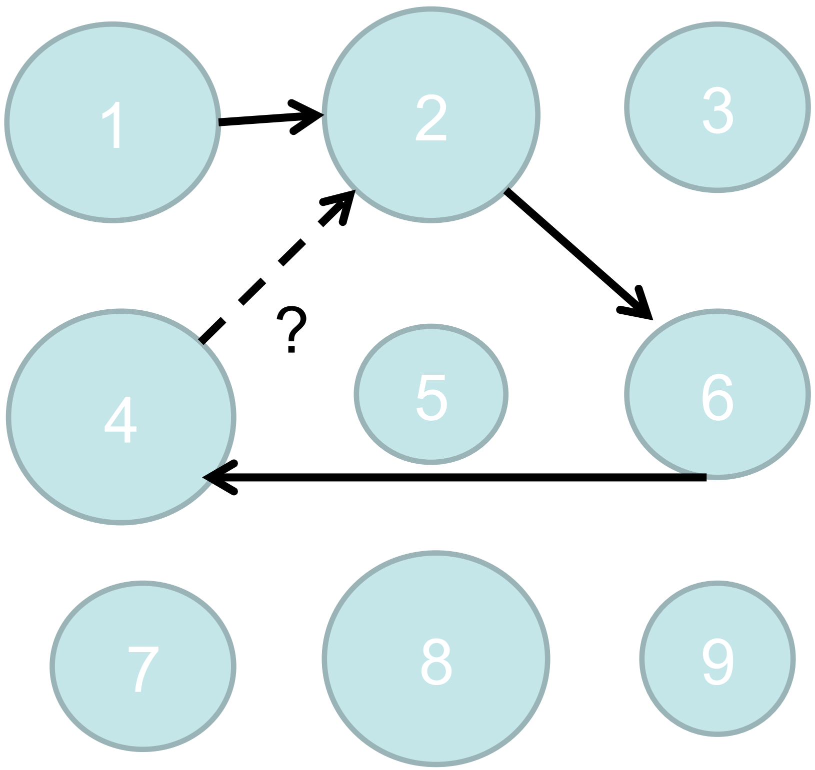
Trapped in two-patterns loop



'adaptation':



Associative retrieval: graph representation



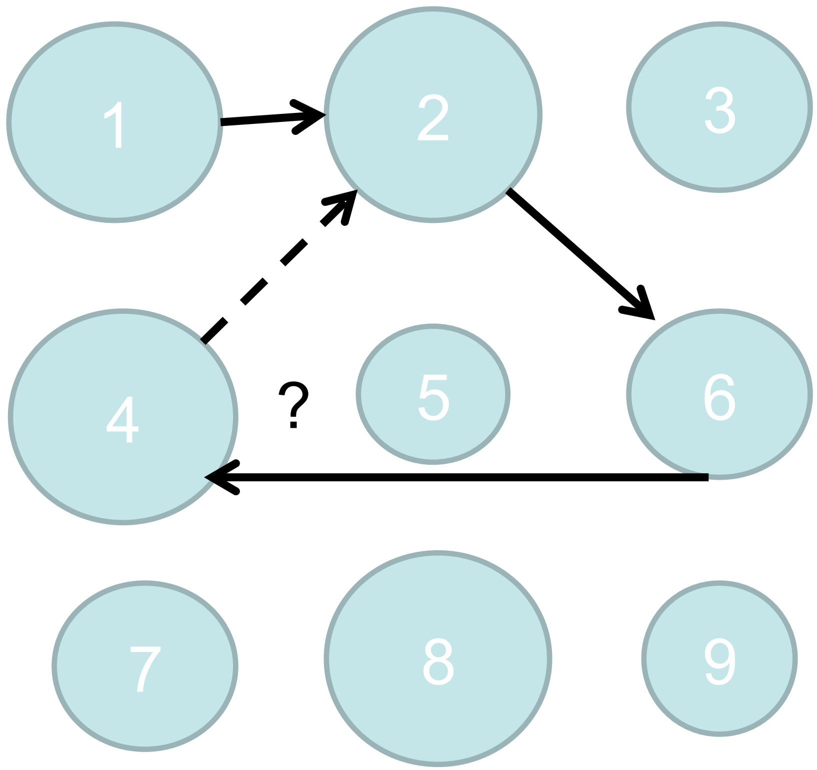
$1 \rightarrow 2$

$2 \rightarrow 6$

$6 \rightarrow 4$

$K \rightarrow K$

Associative retrieval: graph representation



$1 \rightarrow 2$

$2 \rightarrow 6$

$6 \rightarrow 4$

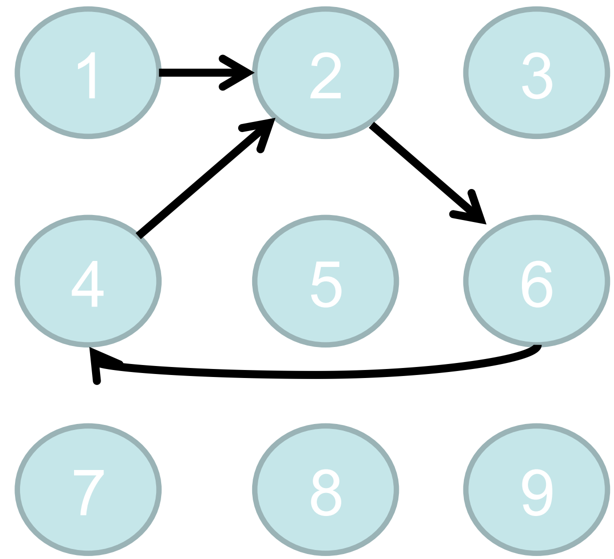
$K \rightarrow K$

$$< Nwr >: \; L^{\alpha} \quad \alpha = \frac{1}{2}\frac{1-f}{1+f}$$

Romani et al, 2013

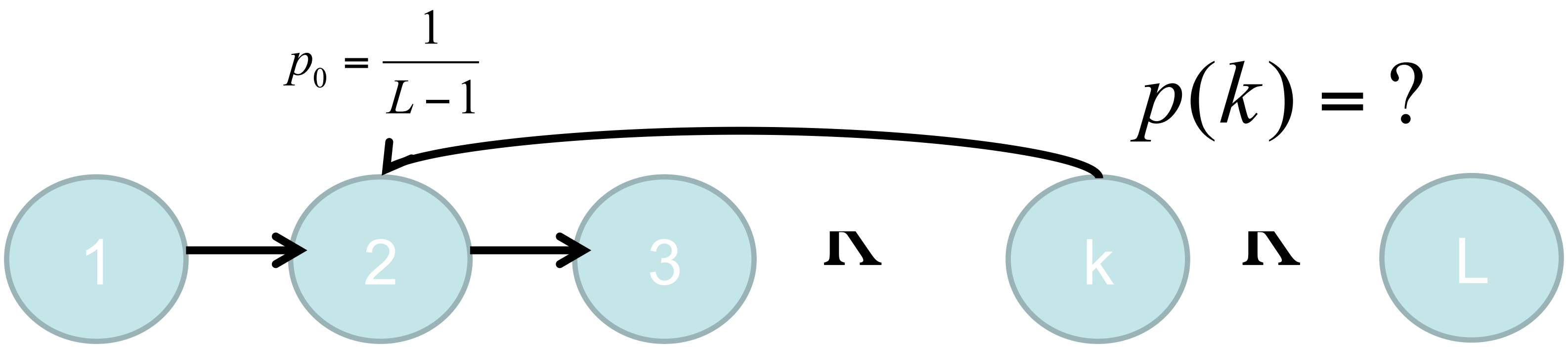
Random asymmetric matrix of similarities: exact solution of the model

$$\begin{pmatrix} \times & S_{1,2} & \cdots & S_{1,L} \\ S_{2,1} & \times & \cdots & S_{2,L} \\ \vdots & & \ddots & \vdots \\ S_{L,1} & S_{L,2} & \cdots & \times \end{pmatrix} \Rightarrow$$



All S independent iid → All possible arrows are equally probable

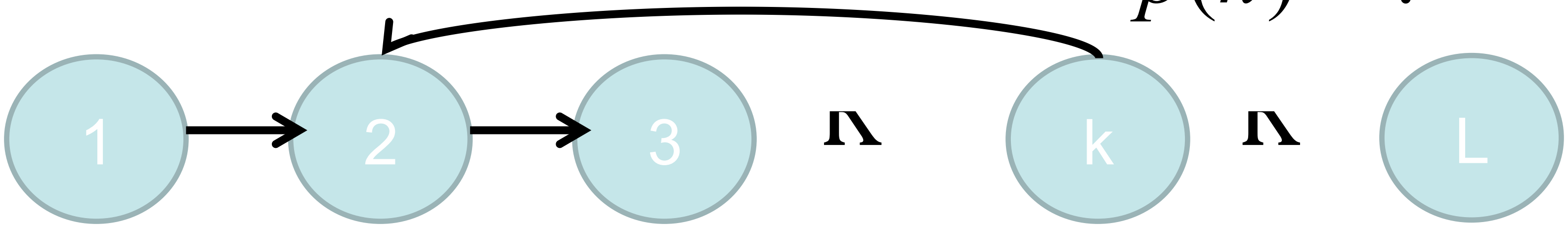
Random asymmetric matrix of similarities: exact solution of the model



Random asymmetric matrix of similarities: exact solution of the model

$$p_0 = \frac{1}{L-1}$$

$$p(k) = ?$$

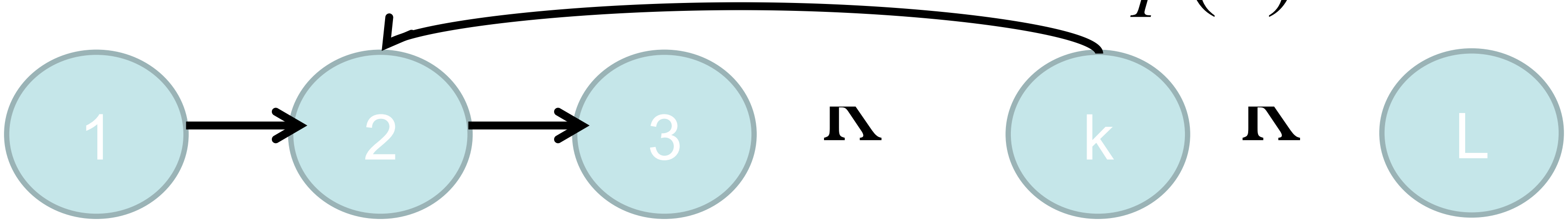


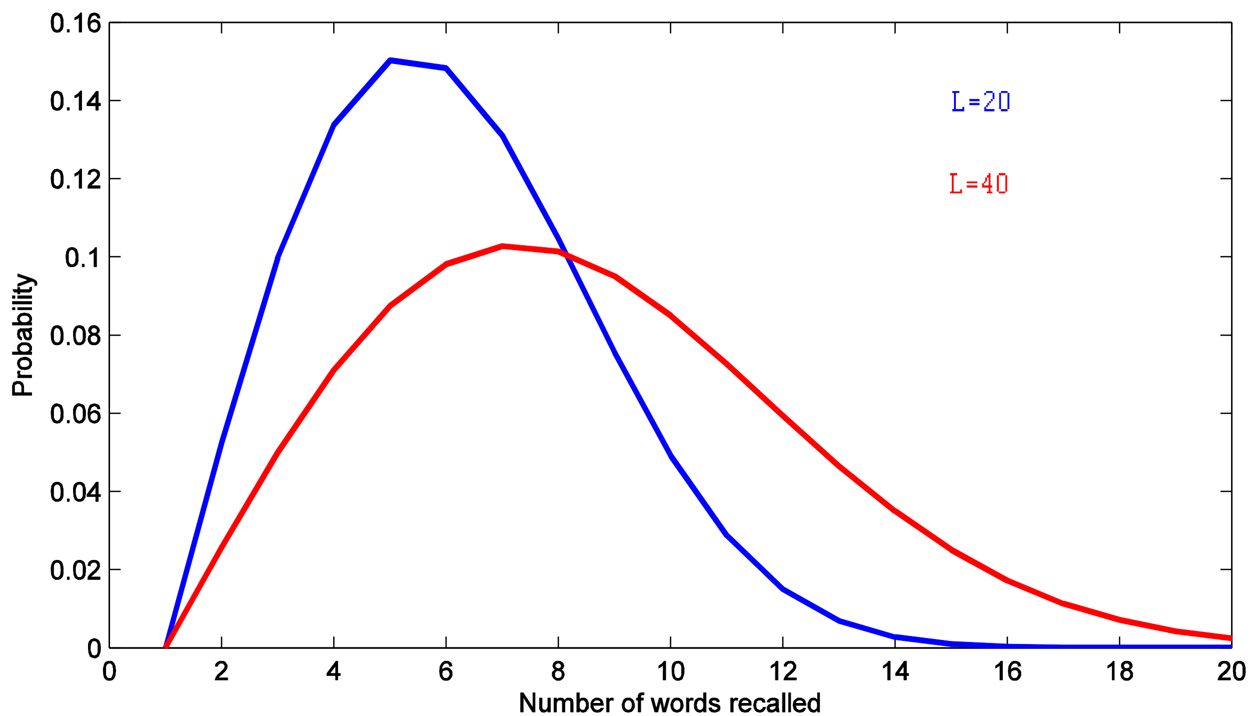
$$p(k) = \left(1 - \frac{1}{L-1}\right)\left(1 - \frac{2}{L-1}\right)\ldots\left(1 - \frac{k-2}{L-1}\right)\frac{k-1}{L-1}$$

Random asymmetric matrix of similarities: exact solution of the model

$$p_0 = \frac{1}{L-1}$$

$$p(k) = ?$$

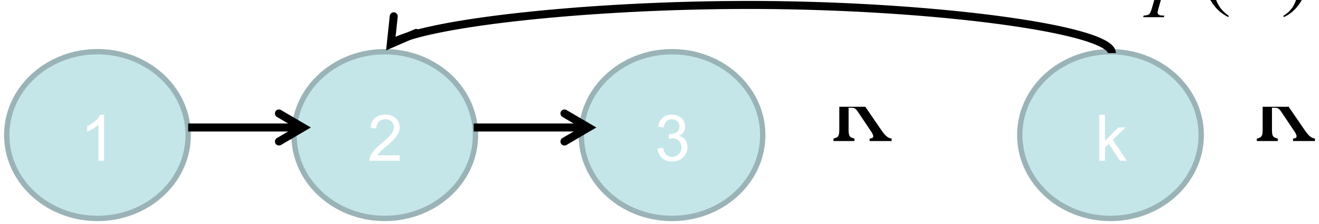




Power law scaling

$$p_0 = \frac{1}{L-1}$$

$$p(k) = ?$$



$$p(k) = (1-\frac{1}{L-1})(1-\frac{2}{L-1})\ldots(1-\frac{k-1}{L-1})\frac{k}{L-1}$$

$$k << L$$

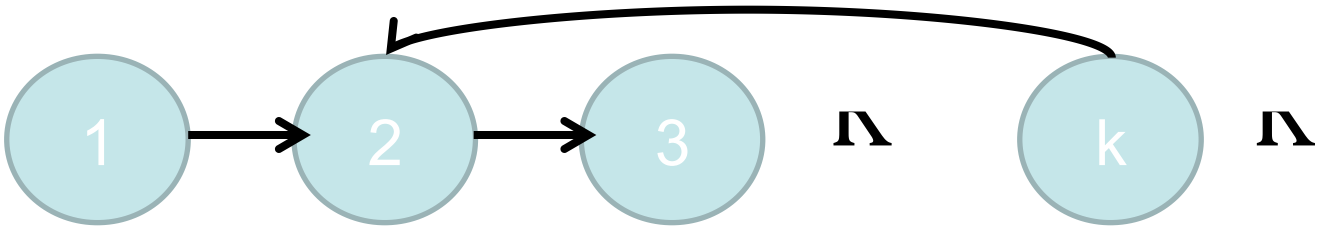
$$\approx \frac{k}{L}\exp\left(-\frac{1}{L}\sum_{i=1}^{k} i\right) \approx \frac{k}{L}\exp\left(-\frac{k^2}{2L}\right)$$

$$x = \frac{k}{\sqrt{L}} \Rightarrow p(x) = x\exp(-\frac{x^2}{2})$$

Power law retrieval capacity

$$p_0 = \frac{1}{L-1}$$

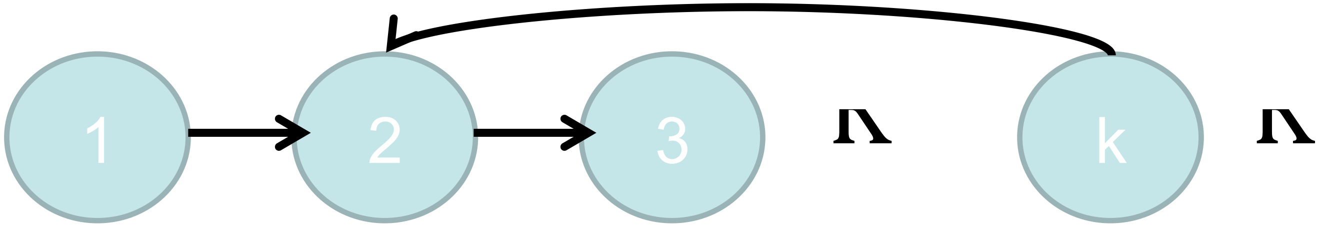
$$p(k) = ?$$



$$x = \frac{k}{\sqrt{L}} \Rightarrow p(x) = x \exp\left(-\frac{x^2}{2}\right)$$

Power law retrieval capacity

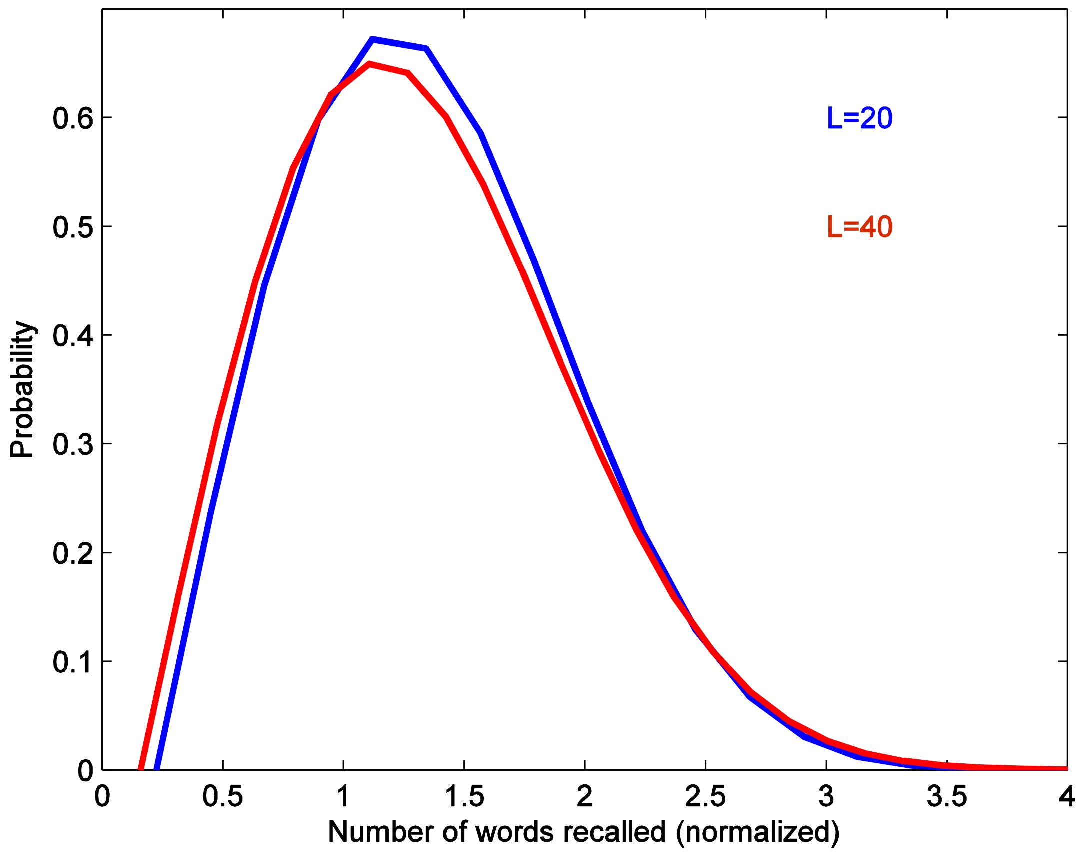
$$p_0 = \frac{1}{L-1} \qquad p(k) = ?$$



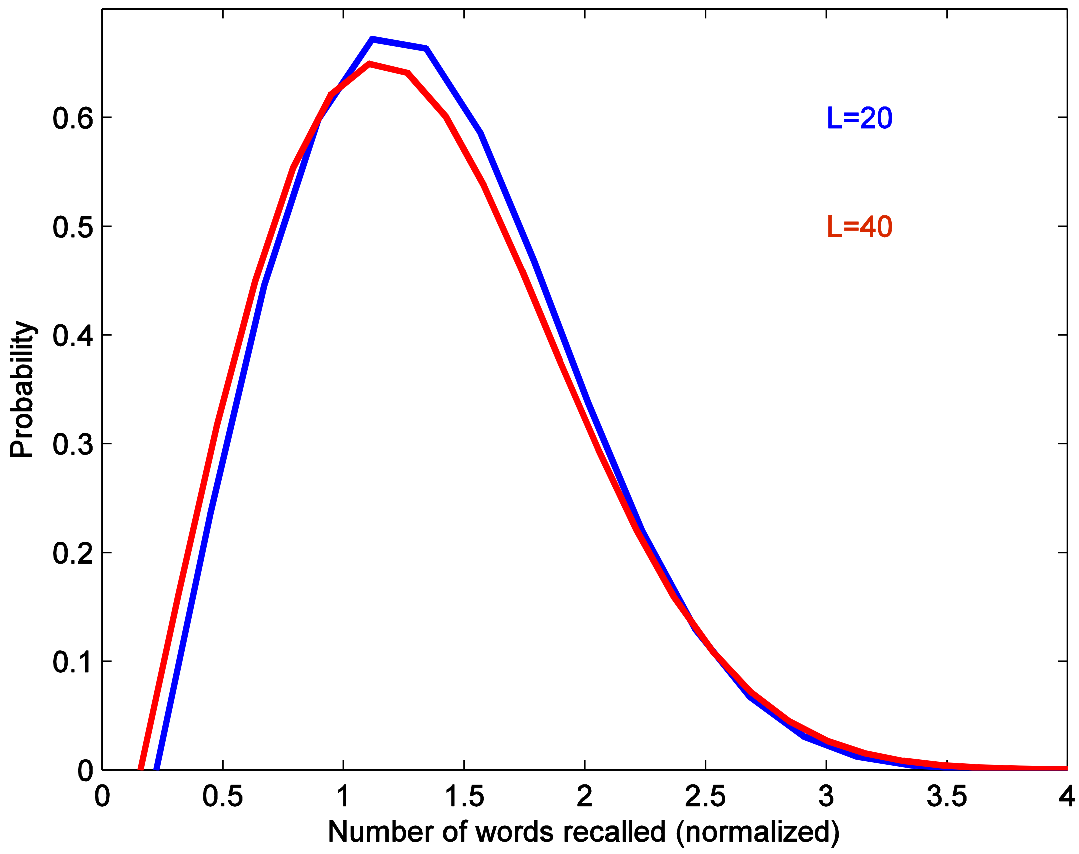
$$x = \frac{k}{\sqrt{L}} \Rightarrow p(x) = x \exp(-\frac{x^2}{2})$$

$$p(k) = \frac{k}{L} \exp(-\frac{k^2}{L})$$

Normalized probability distribution



Normalized probability distribution

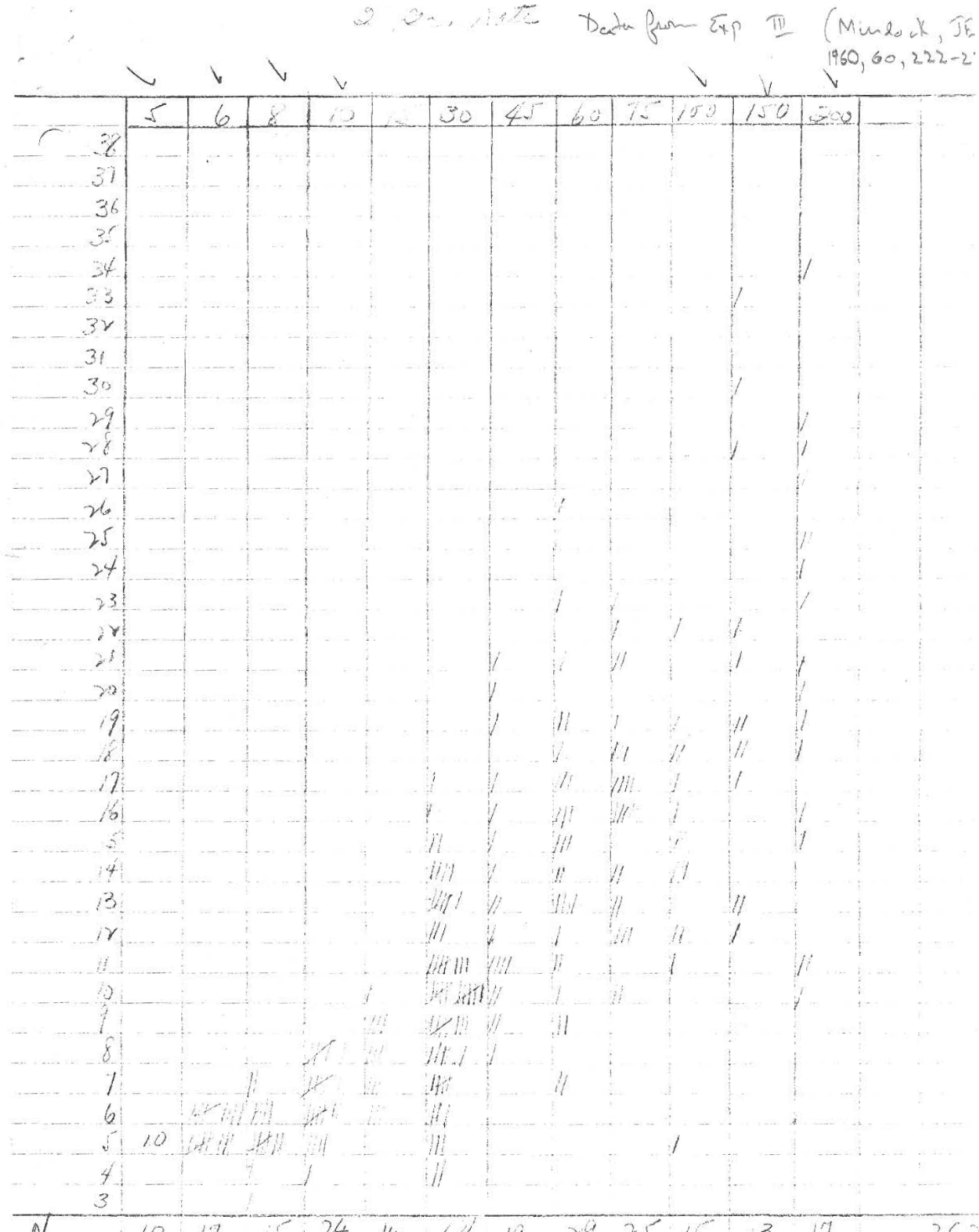


$$\langle k \rangle \approx \sqrt{\frac{\pi}{2}} \cdot L^{1/2}$$

$$Var(k) \approx (2 - \frac{\pi}{2})L$$

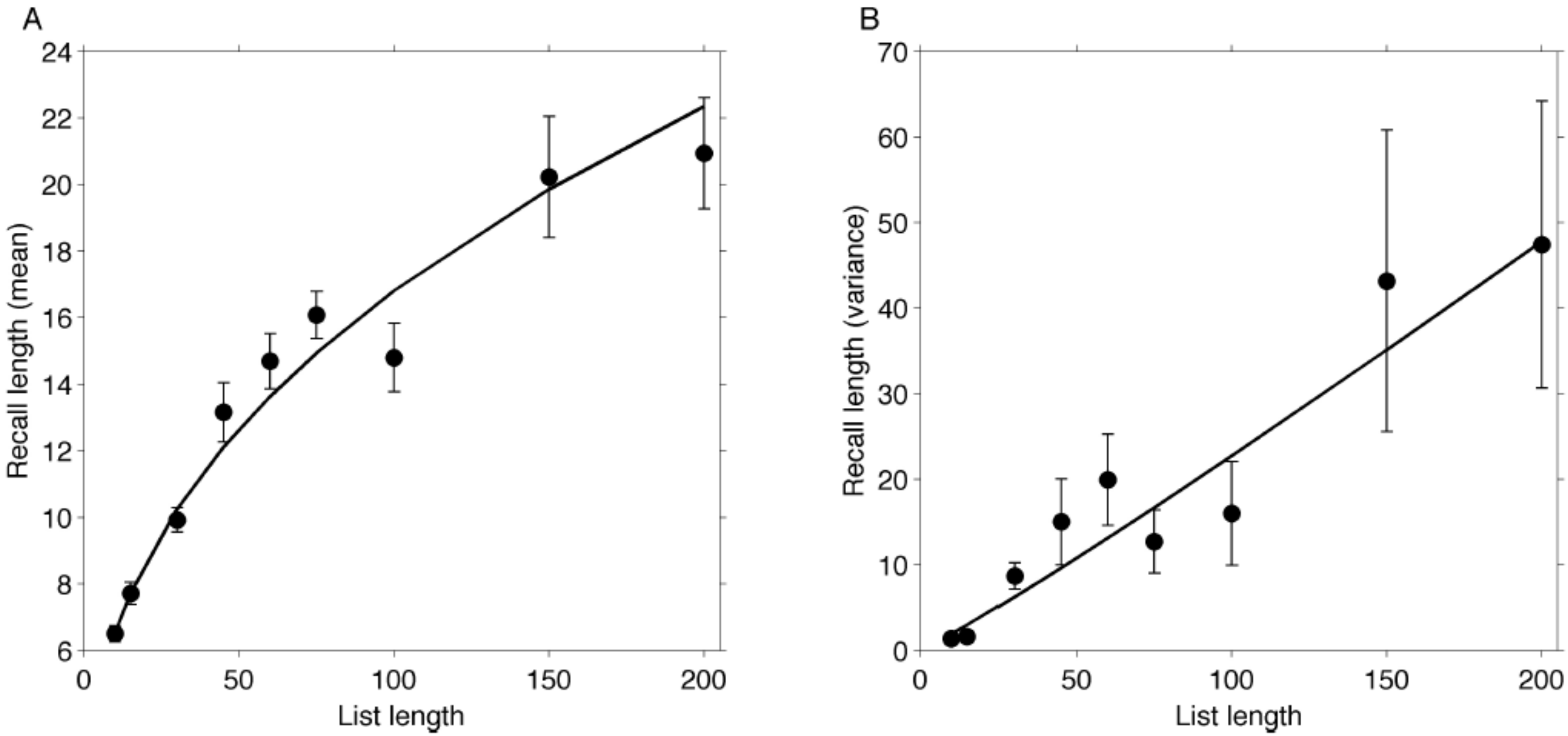
Bennet Murdock
(Toronto)

Retrieval capacity: longer lists

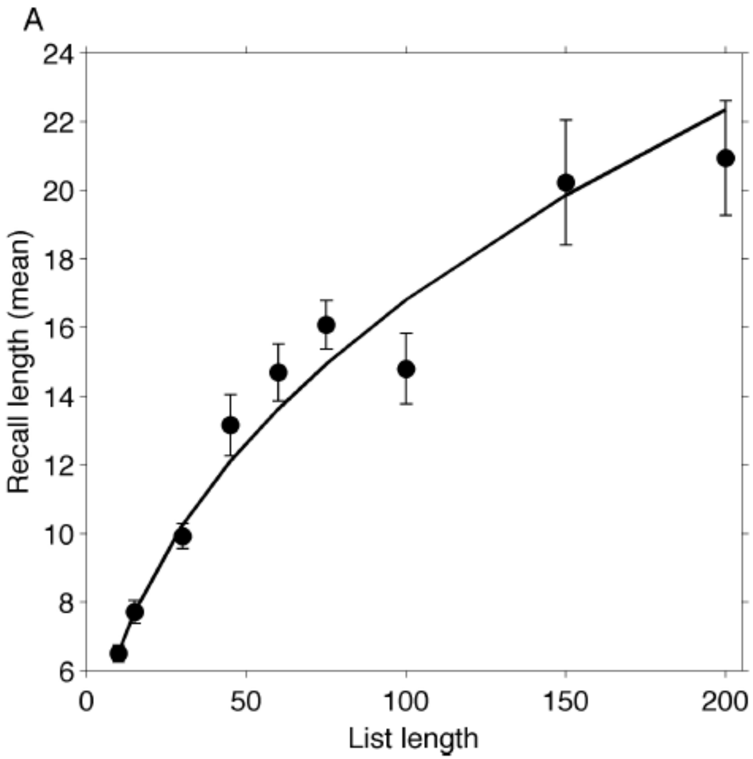


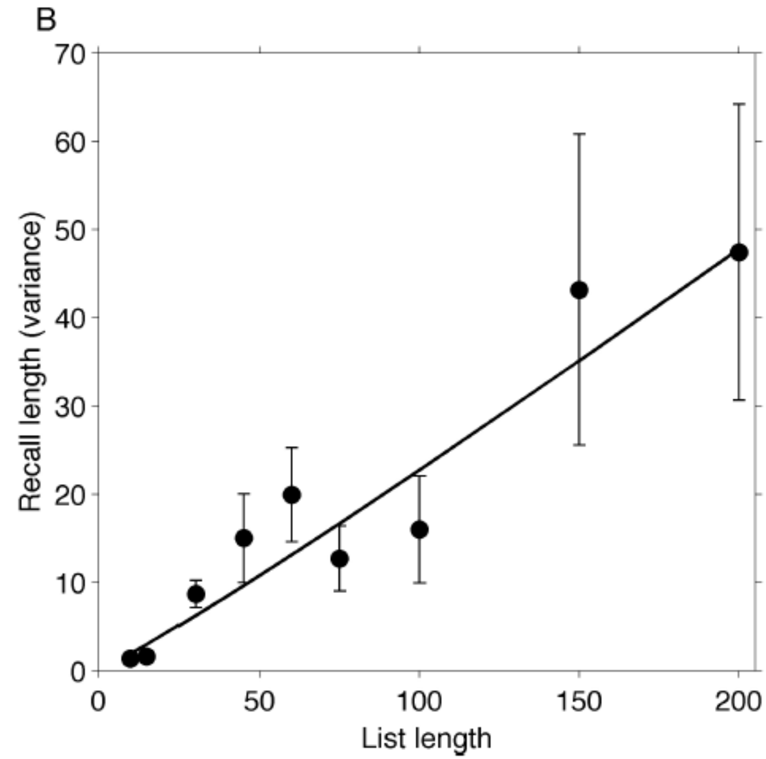
Courtesy of B. Murdock

Retrieval capacity: longer lists



Retrieval capacity: longer lists





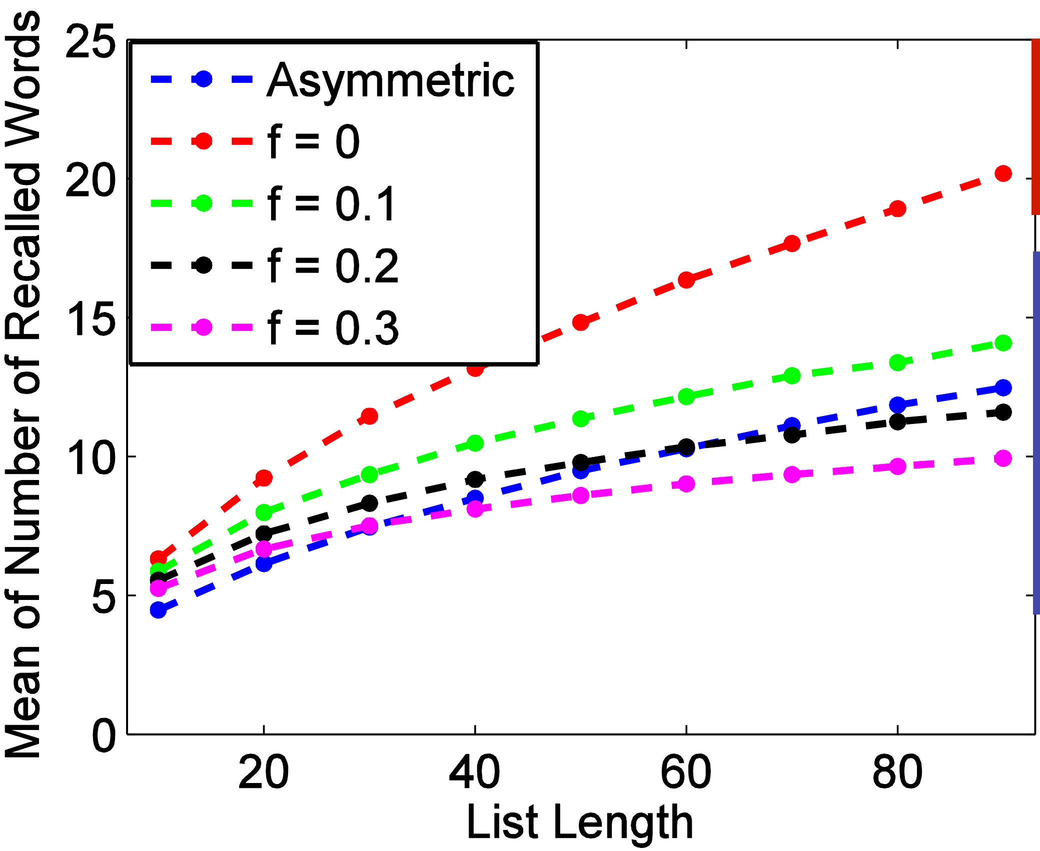
$$\langle k \rangle : \; L^{0.41}$$

$$Var(k) : \; L^{1.08}$$

Conclusions from the simplified model

- Accounts for power law scaling of retrieval capacity
- Predicts nontrivial scaling of the variance of the number of retrieved words
- No free parameters! (All subjects, all memory items are equivalent).

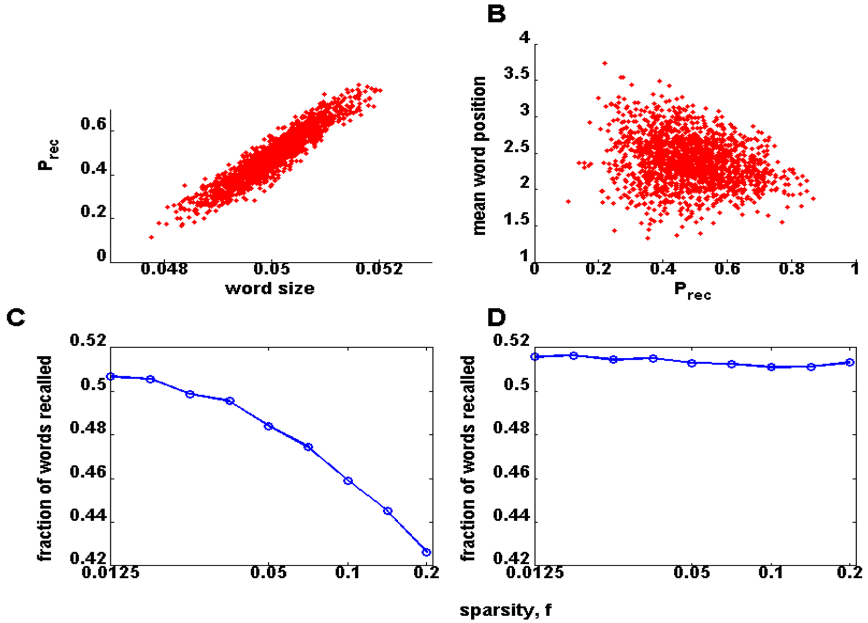
Simulations



$$< Nwr > \approx 2\sqrt{L}$$

$$< Nwr > \approx \sqrt{\frac{\pi}{2} L}$$

‘Easy’ vs ‘difficult’ words



Katkov et al, 2015

Free recall data set

170 subjects

112 trials/ 6 sessions for each subject

16 words in each presented list, no list repetitions.

Immediate recall

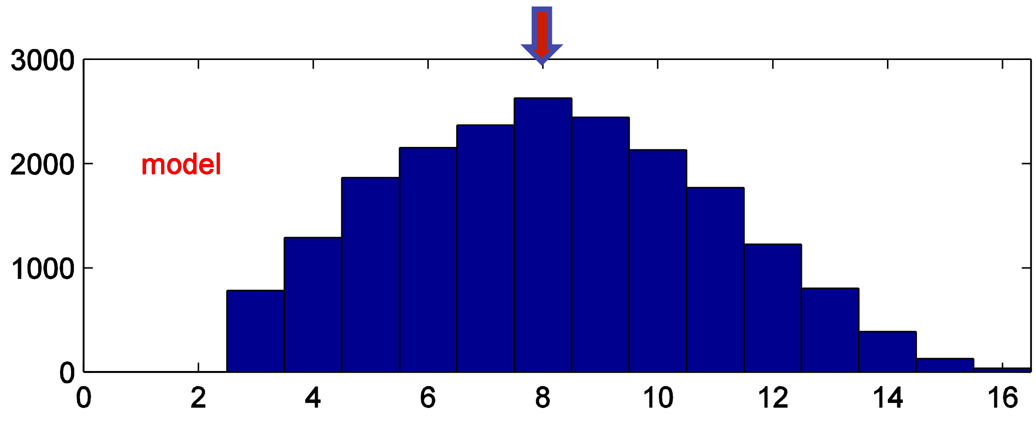
Mike Kahana
Upenn

Retrieval capacity: analytical solution

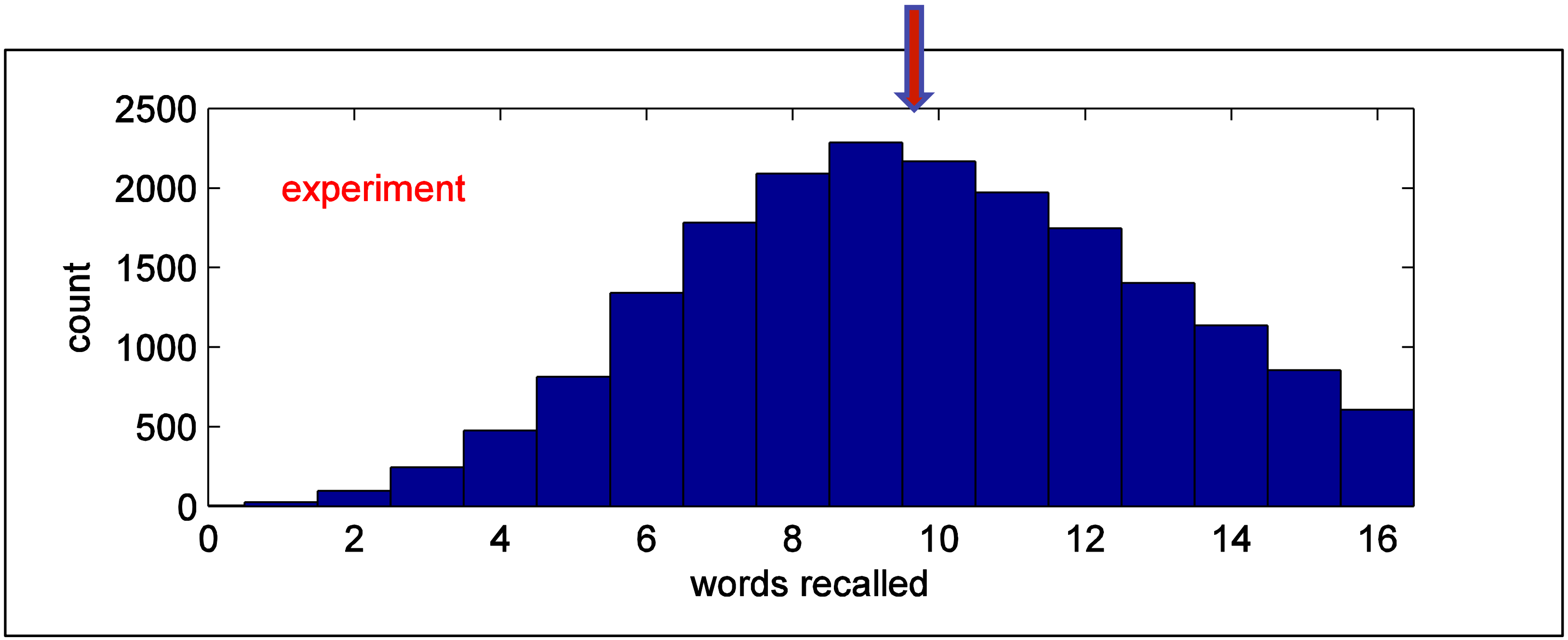
$$f = 1 \qquad < Nwr > \approx 2\sqrt{L}$$

Romani et al 2013

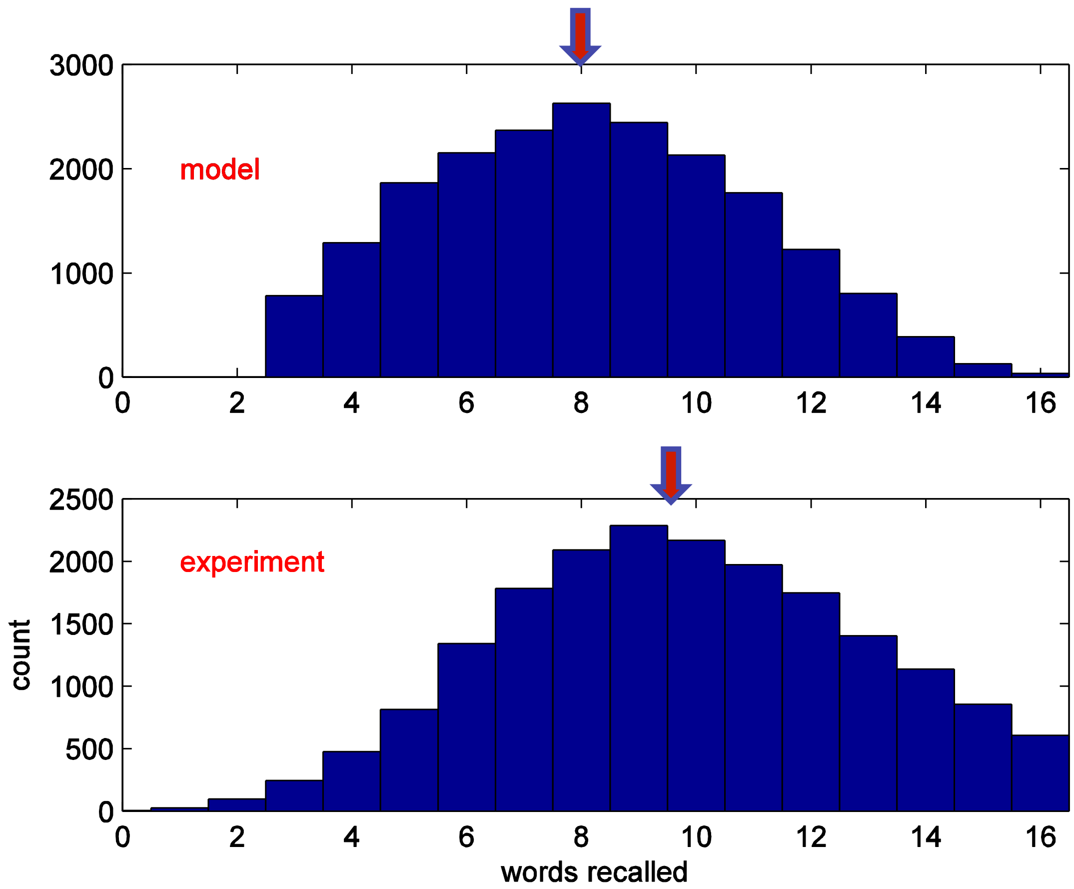
Distribution of recall performance across trials



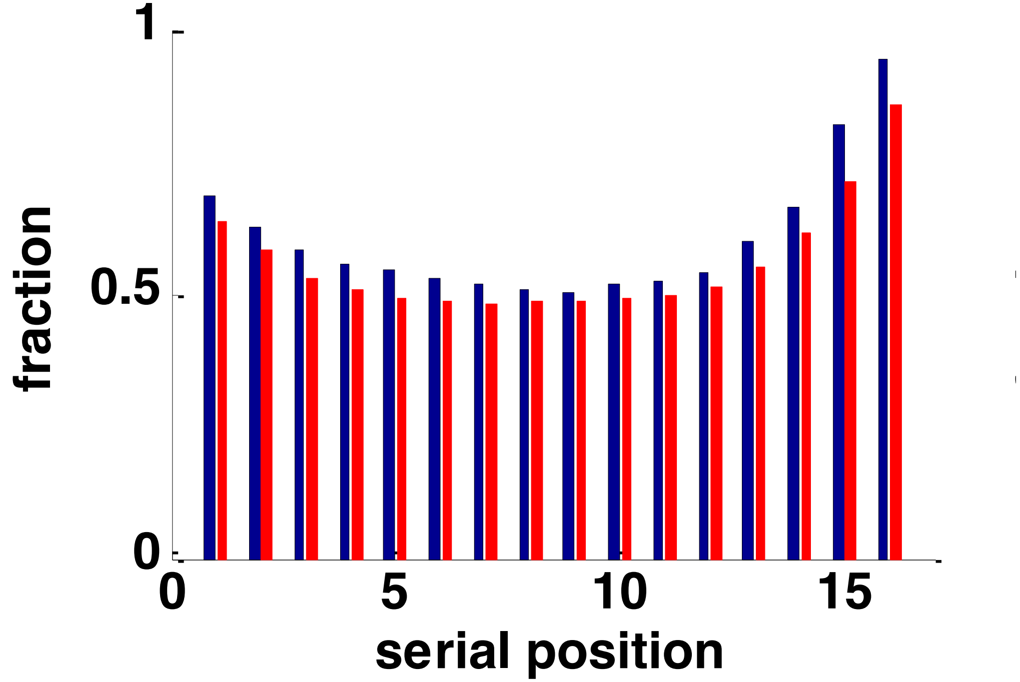
Distribution of recall performance across trials



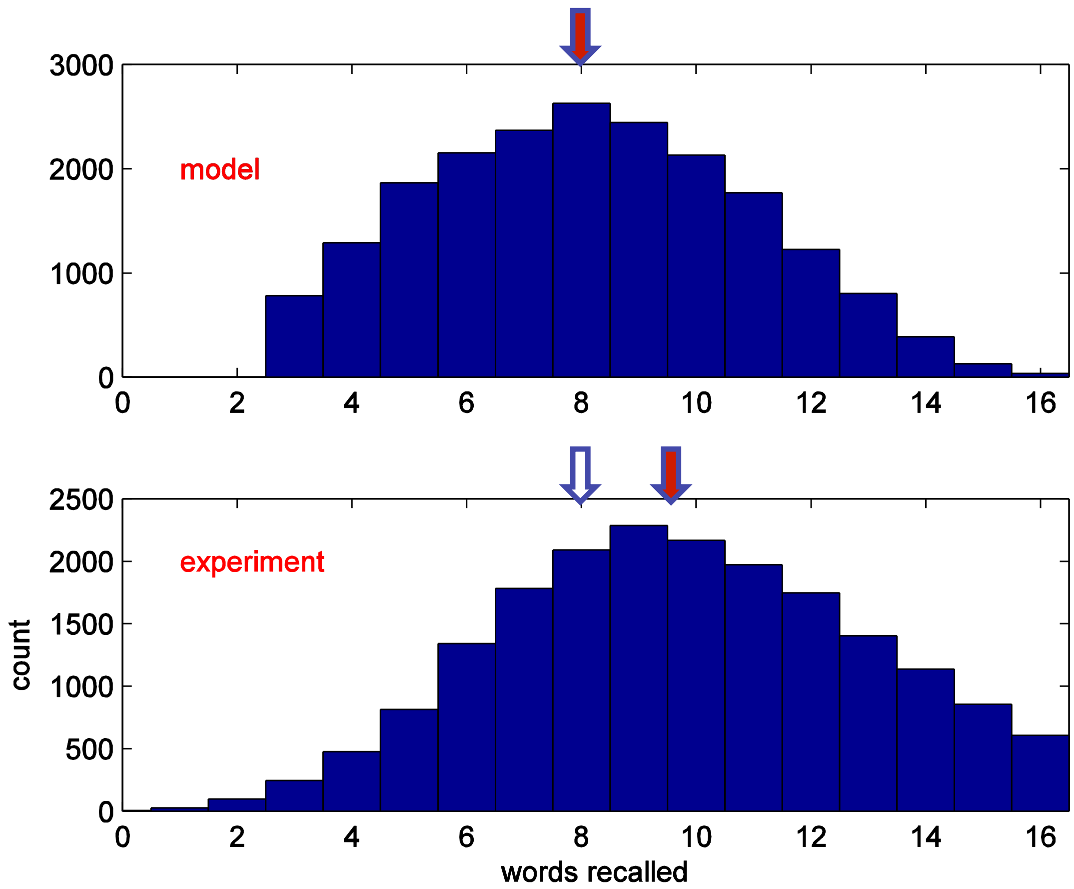
Distribution of recall performance: data vs model



Recency (working memory)

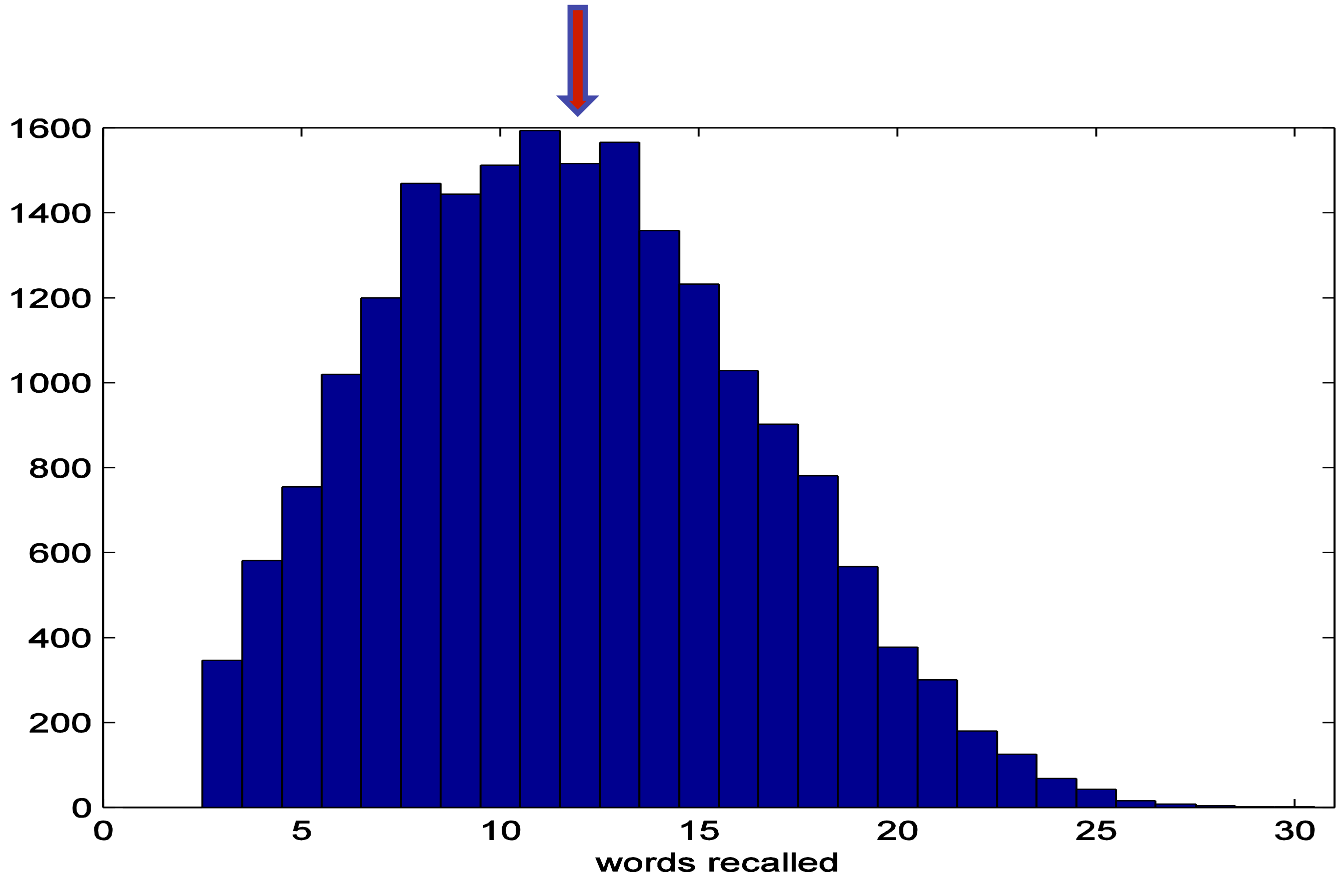


Distribution of recall performance: data vs model

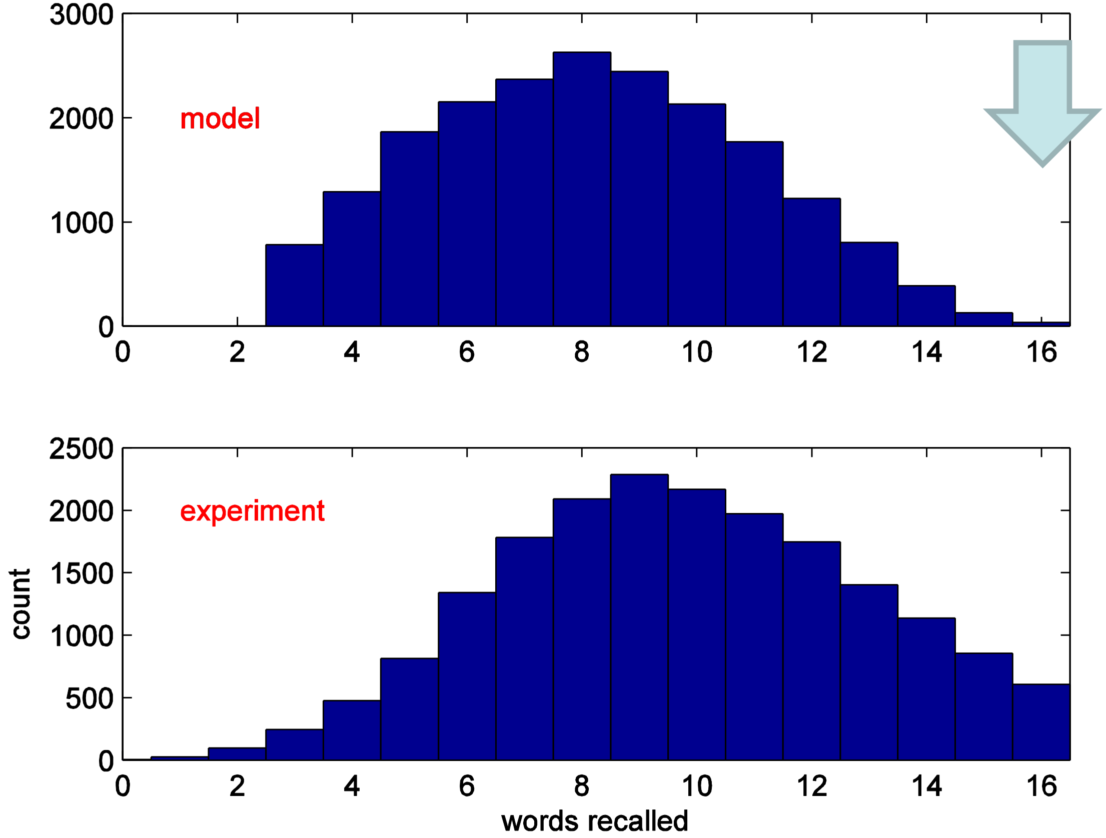


Without recency

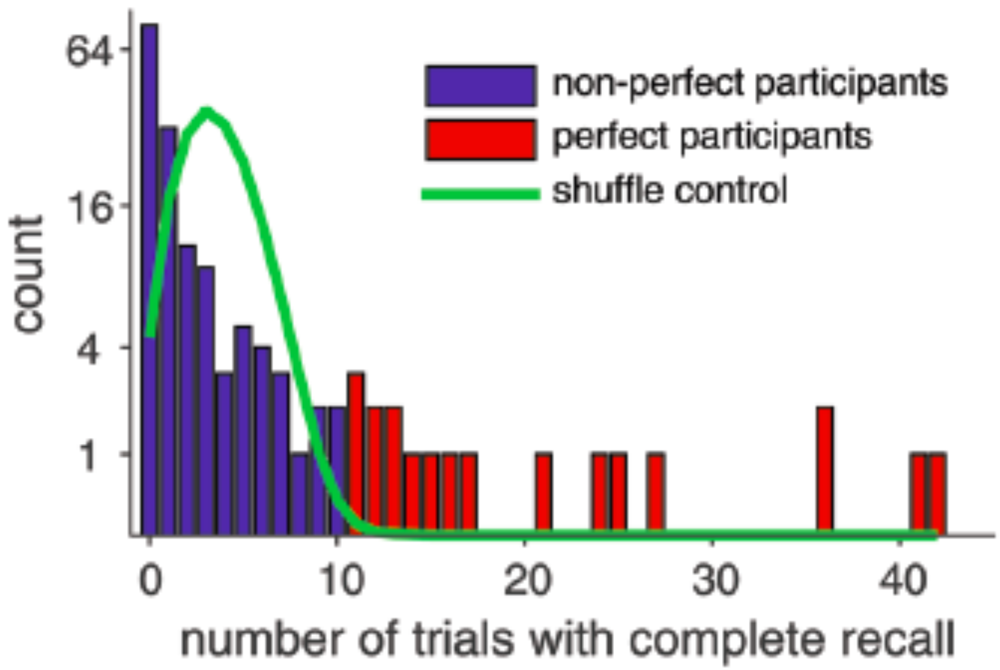
Prediction for L=32 experiment



Distribution of recall performance: data vs model

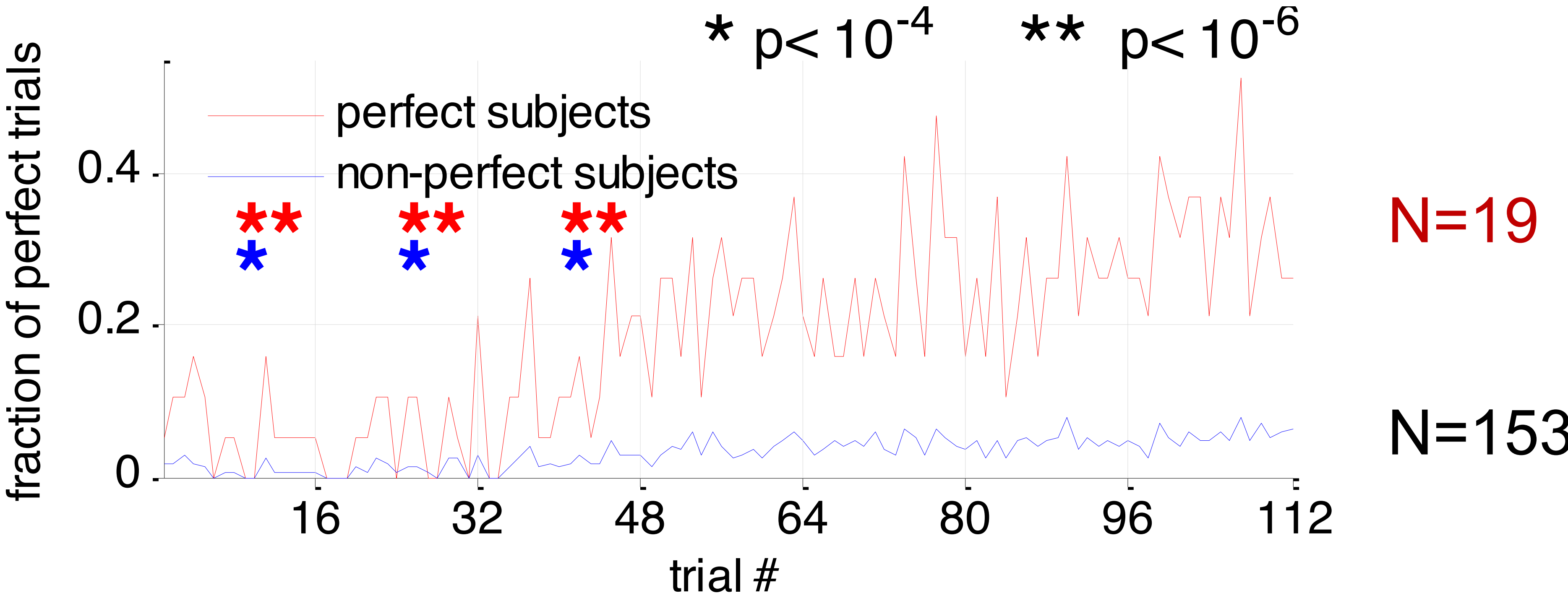


Perfect trials and perfect participants



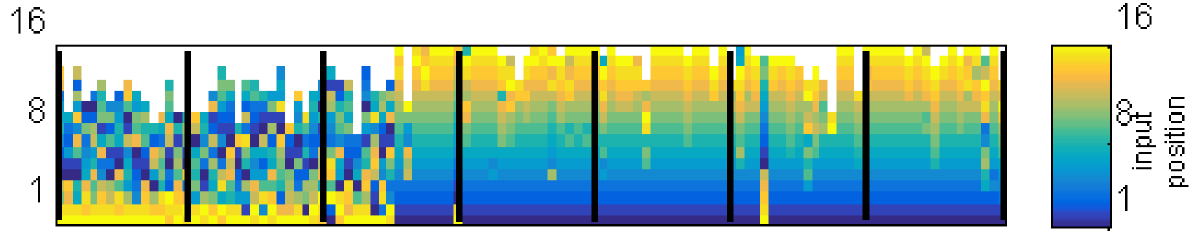
Romani, Katkov and Tsodyks, 2016

Learning to remember: perfect trials and perfect participants

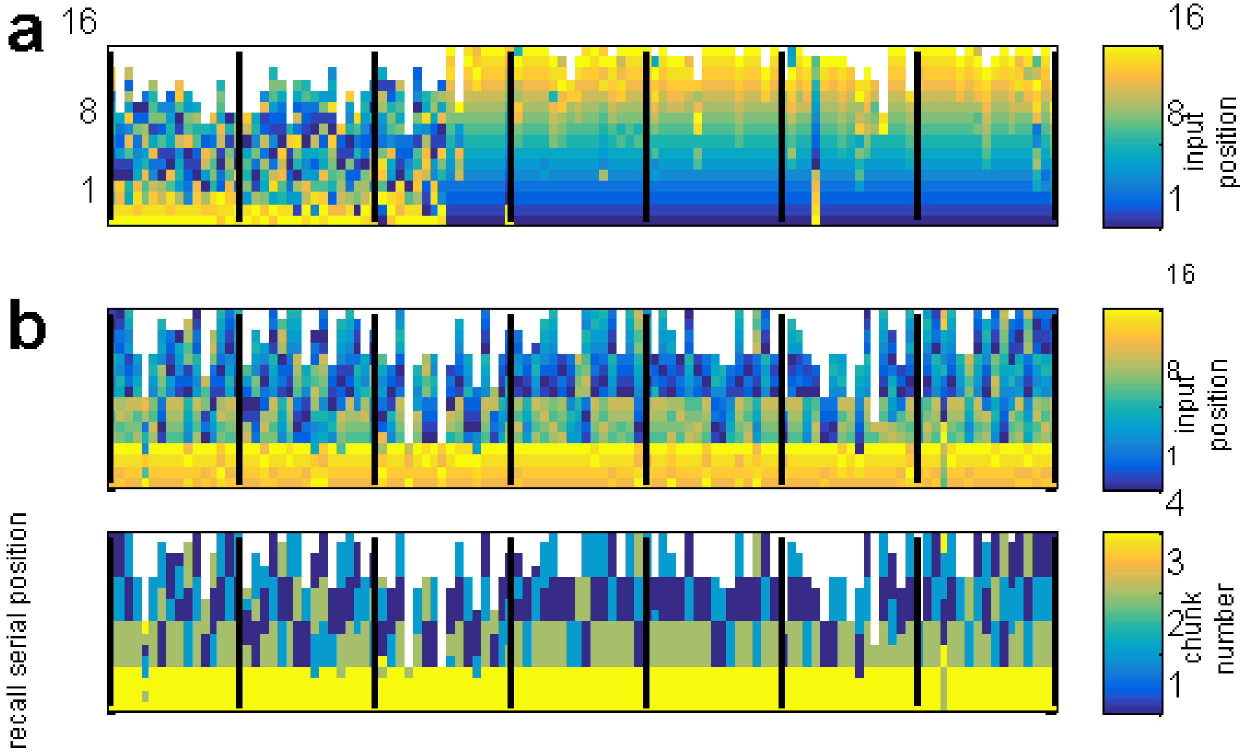


Three exceptional perfect participants

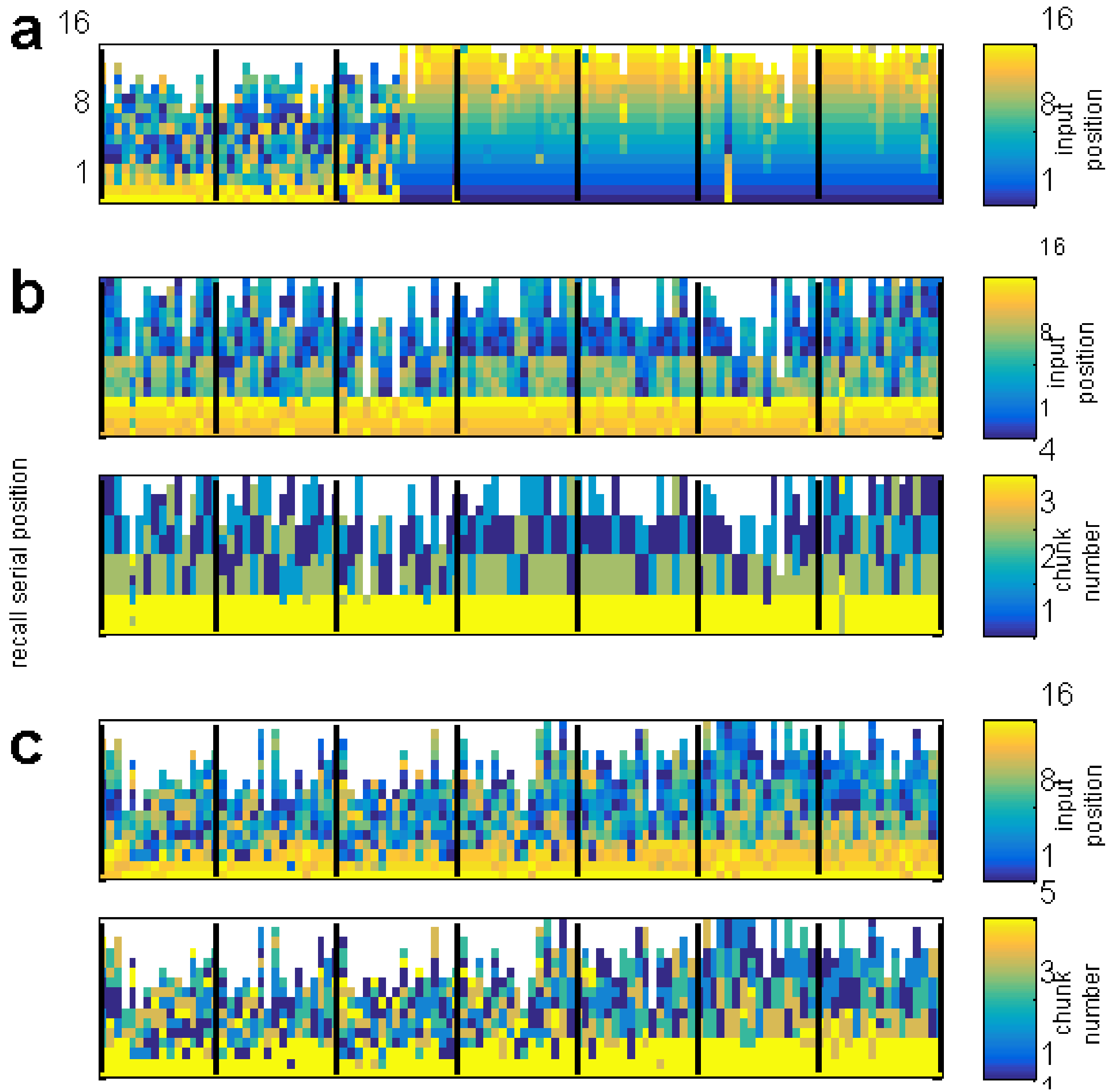
a



Three exceptional perfect participants



Three exceptional perfect participants



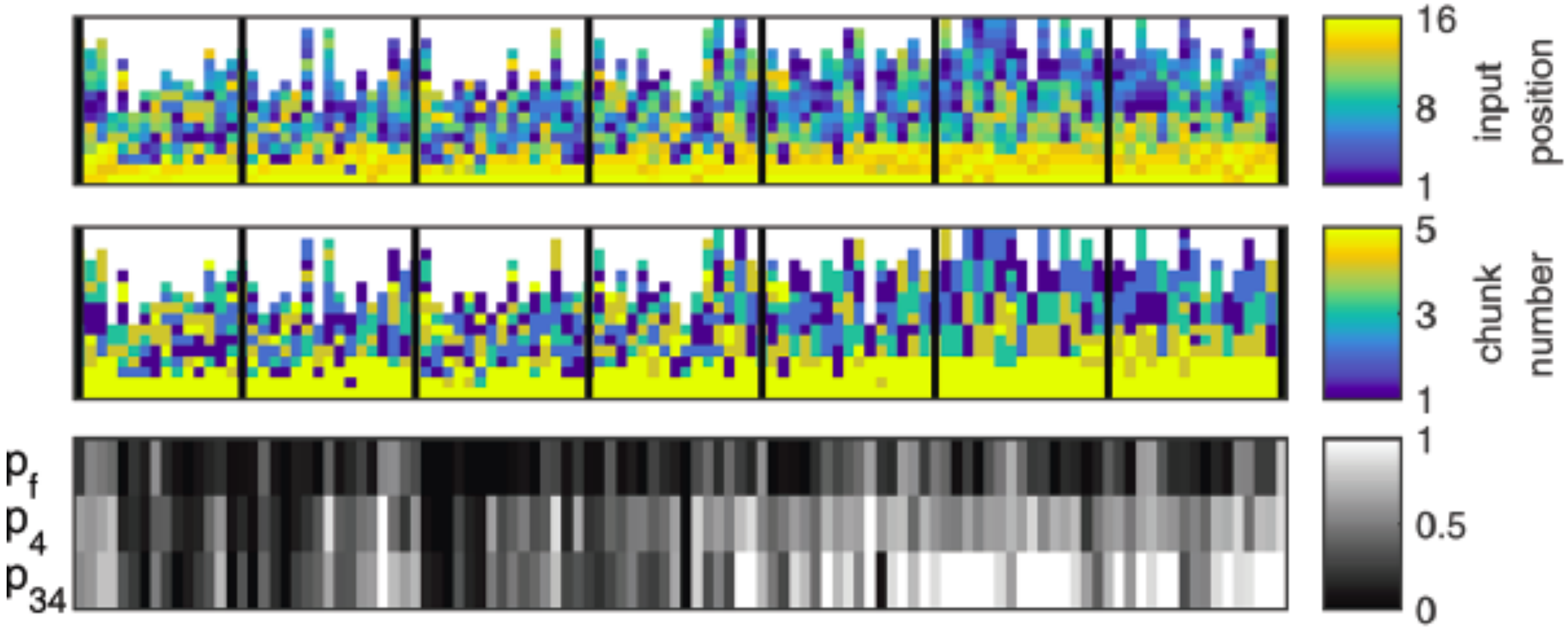
Recall strategies

- Forward chaining
- Chunking4: 4-4-4-4
- Chunking34: 3-3-3-3-4

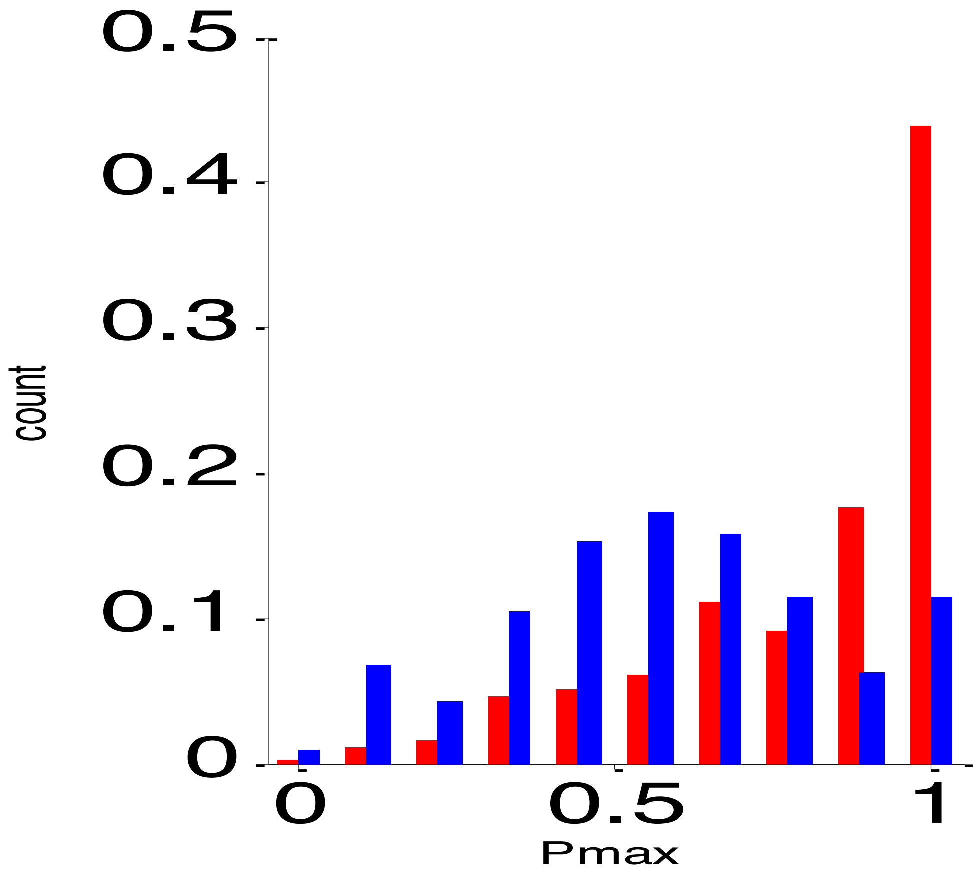
Single trial measure of strategy application

$$(p_f, p_4, p_{34}) \rightarrow \frac{p_{\max}}{strategy}$$

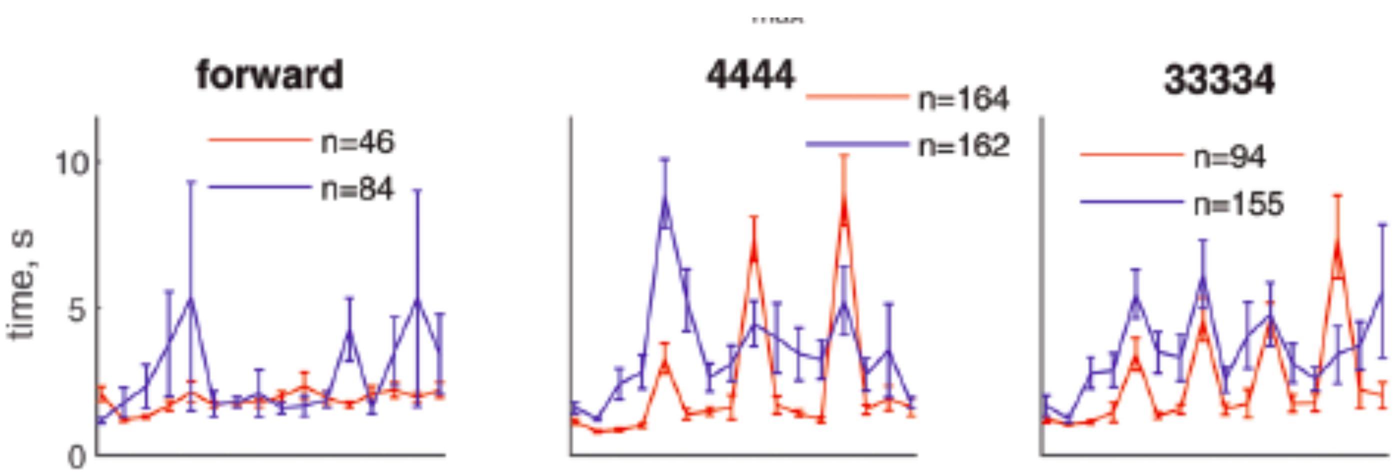
Single trial measure
of strategy application



Perfect vs non-perfect participants: perfect trials

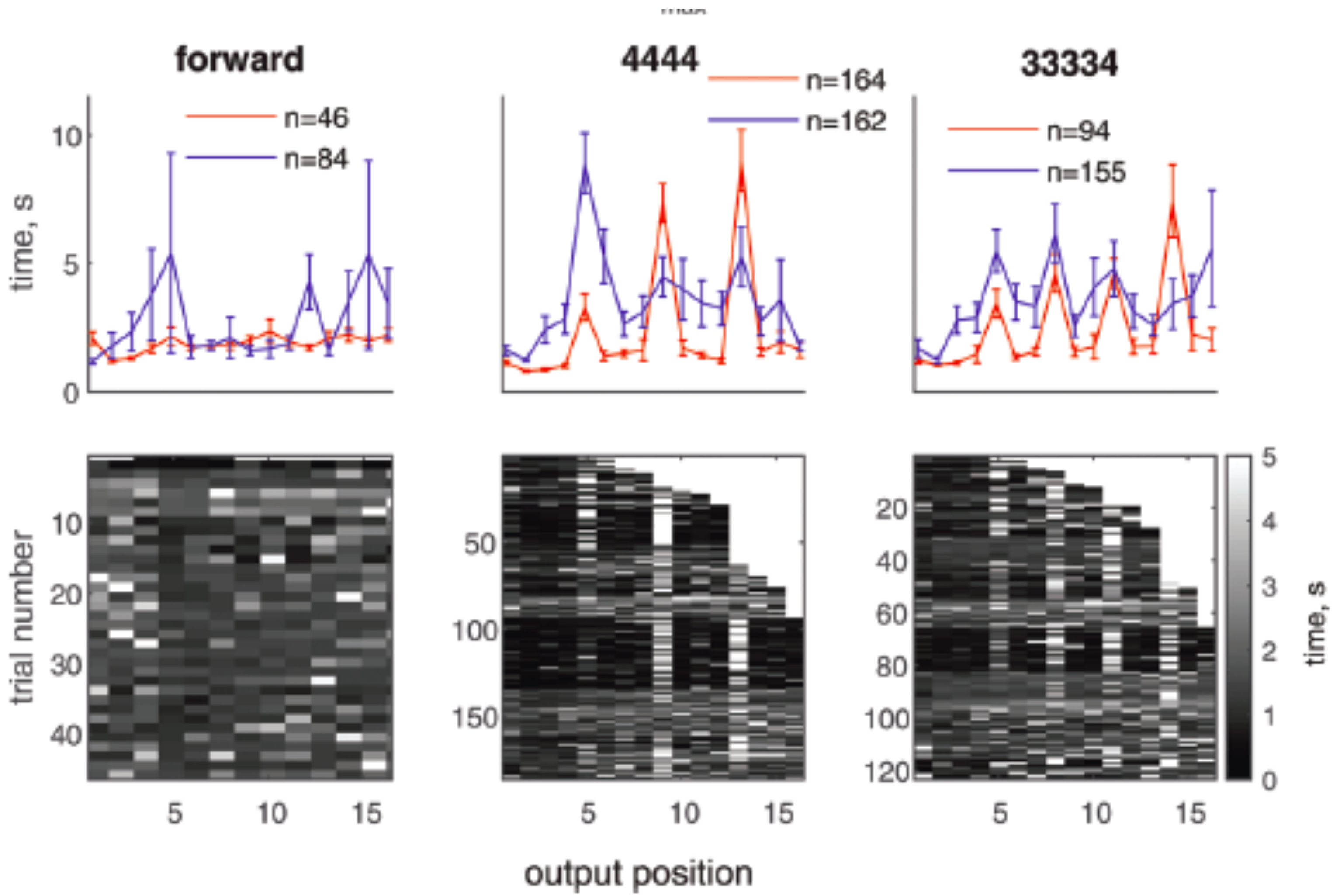


Recall strategies and recall speed: trials with $p_{\max} = 1$

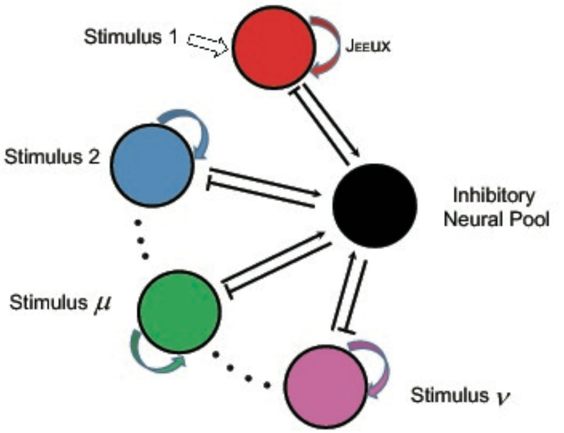


Recall position

Recall strategies and recall speed: trials with $p_{\max} = 1$



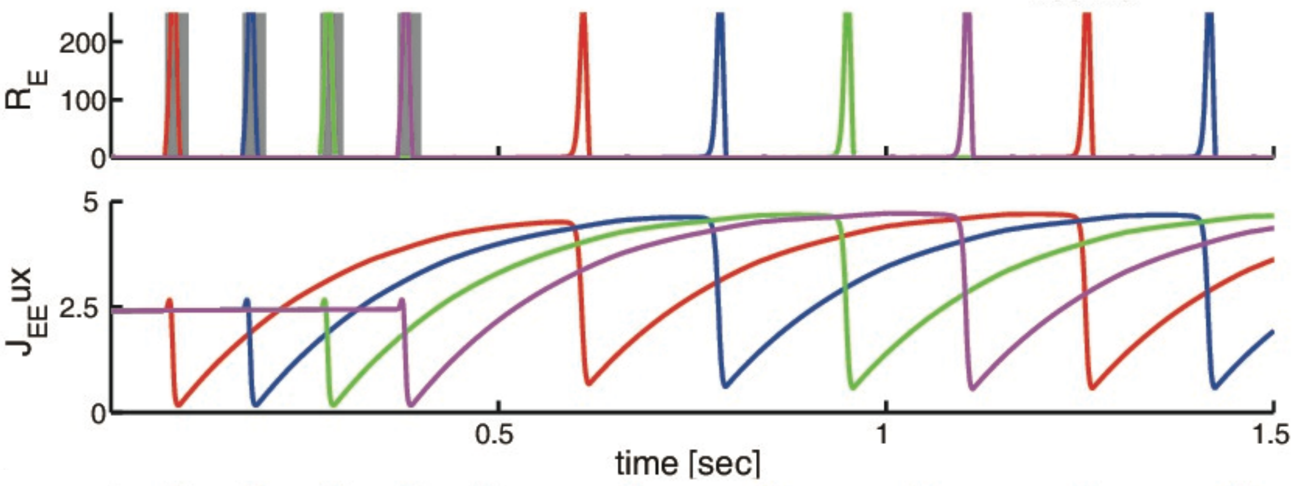
Chunking and working memory capacity (if time permits)



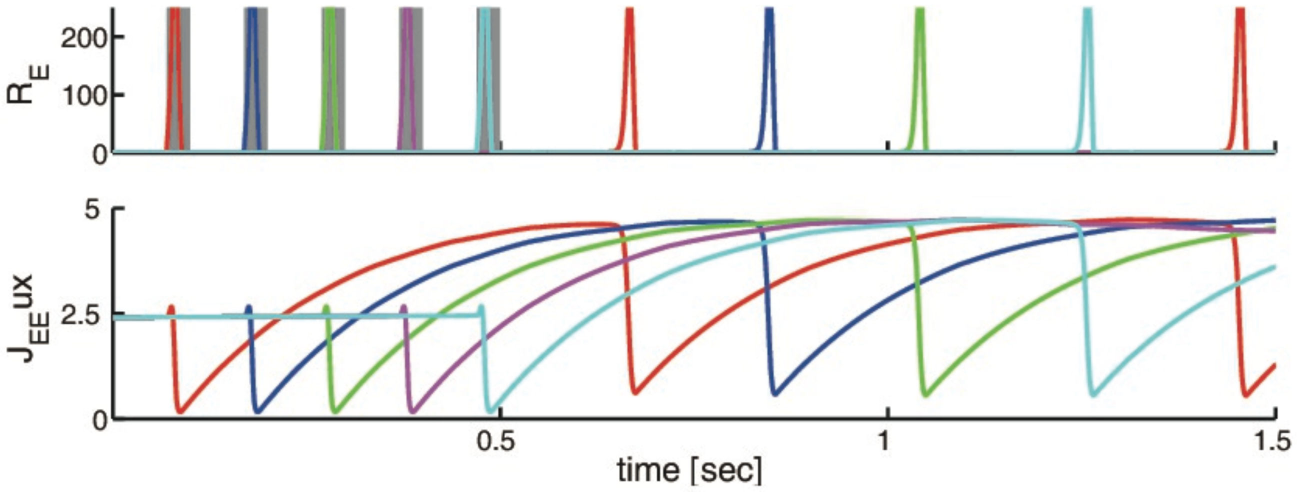
- ✓ Each memory item is stored at an excitatory neural cluster with short-term plasticity in recurrent connections;
- ✓ All excitatory neural clusters are connected to an inhibitory neuron pool;
- ✓ The inhibitory neuron pool feedback to all excitatory clusters, inducing competition among excitatory clusters.

Mongillo et al, 2008; Mi, Tsodyks, submitted

Chunking and working memory capacity



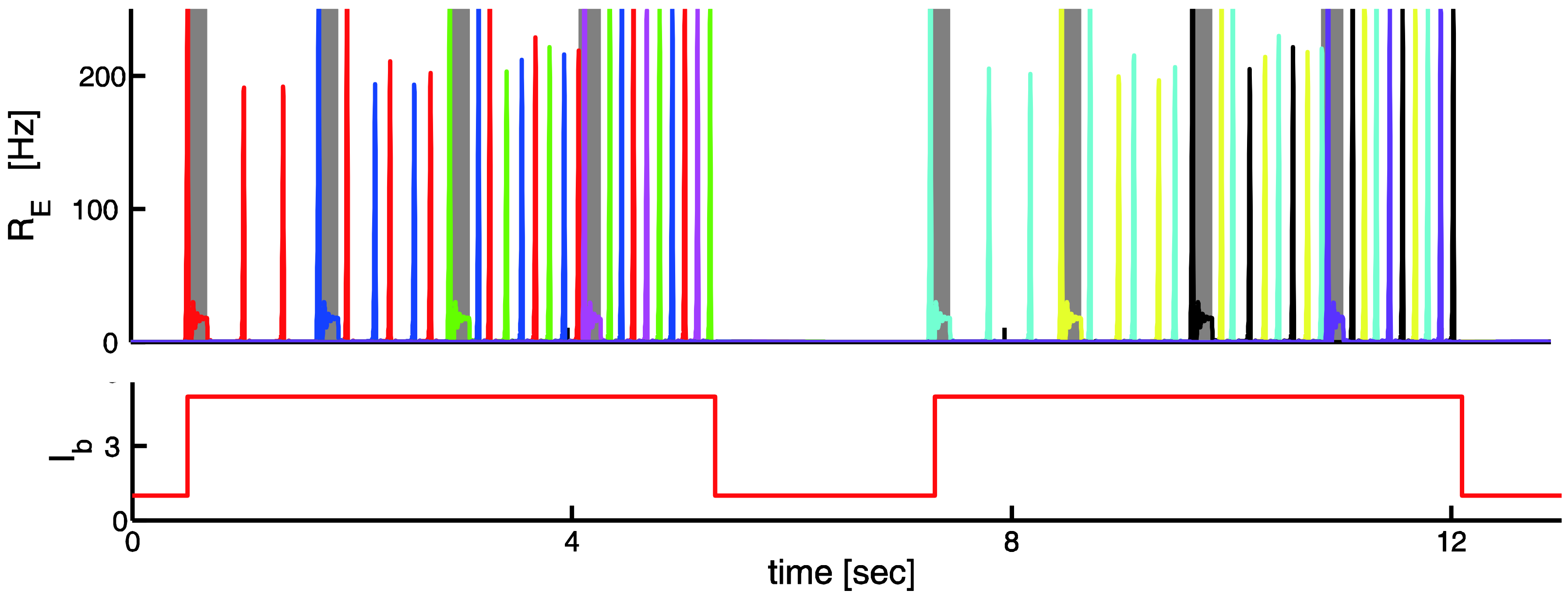
Chunking and working memory capacity



$$C \sim \frac{\tau_d}{\tau} \frac{\log \dfrac{\tau_f / \tau_d}{1-U}}{\log \dfrac{|h_0|}{I_{background} - I_{crit}}}$$

(Mi, Tsodyks, submitted)

Emergence of chunking



Failure of chunking